MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Dr. J. L. E. DREYER, PRESIDENT, in the Chair.

Rev. Herbert Douglas Anthony, The College, Richmond, Surrey;
K. Chand, B.A., Sidney Sussex College, Cambridge;
E. C. Isaac, 47 Apuka Road, Brooklyn, Wellington, New Zealand;
Rev. William Lipscombe Orpwood, St Matthew's Vicarage, Hurlingham, London, S.W.;
Rev. Arthur Maitland Peckham, F.Z.S., 28 Hornsey Rise, London, N. 19;
Harold Woodgate Raisin, B.Sc., 24 Arnewood Road, Bournemouth; and
Professor Issei Yamamoto, Kyoto University Observatory, Japan,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Captain Thomas S. Angus, late P. & O. S. S. Co., Pitreavie, Loughton, Essex (proposed by H. Purey-Cust);
Commander R. Griffiths Bowen, R.A.N., Naval Staff Office, Fremantle, Western Australia (proposed by A. D. Ross);
Ernest George Constable, National Physical Laboratory, Teddington, and Cobalnest, Holmesdale Road, Hampton Wick, Middlesex (proposed by H. L. P. Jolly);
Captain Ellison Hawks, "Arcetri," Wavertree Nook Road, Liverpool (proposed by T. E. R. Phillips);
John Laver, Mantell Street, Moonee Ponds, near Melbourne, Victoria, Australia (proposed by C. J. Merfield);

Professor George Henry Livens, University College, Cardiff (proposed by H. Spencer Jones);
Miss Marie Hazel Losh, M.A., Ann Arbor, Michigan, U.S.A. (proposed by W. J. Hussey);
Lady Lubbock, Ridlands, Limpsfield, Surrey (proposed by H. H. Turner);
Miss Ellen O'Connor, Research Student at the Imperial College of Science and Technology, S. Kensington, London, S.W. (proposed by A. Fowler);
John Potter, 19 Maldon Road, Colchester (proposed by J. F. Rendell);
David Ross, Peace Observatory, Kincraig, Yarra Glen, Victoria, Australia (proposed by C. J. Merfield); and
Bernard P. Scattergood, M.A., M.I.E.E., F.S.A., Bennett's, Harpenden, Herts (proposed by W. A. Parr).

Seventy-eight presents were announced as having been received since the last meeting, including, amongst others:—

Tycho Brahe, Opera Omnia, tomus 10 (presented by Dr. Dreyer).
Boston, American Academy of Science, Memoirs, vol. 4.
J. de Vos van Steenvijk, Catalogue of the Declinations of 2840 southern stars for the equinox of 1825 observed at Paramatta, N.S.W., by James Dunlop and Charles Rümker in the years 1823–1828. Re-reduced.
J. F. Schroeter, Haandbog i Kronologi.

Mr. W. A. Parr, Mr. R. T. Cullen and Mr. W. M. H. Greaves were appointed auditors of the Treasurer's accounts for 1923.

Internal Motions in Spiral Nebulæ.

By J. H. Jeans, Sec. R.S.

1. No interpretation, and very little discussion, has so far appeared of Mr. Van Maanen's highly interesting measurements of the internal motions of spiral nebulæ. From his statement * that for lack of suitable material no further measurements can be made for some years, it would appear that the present time is opportune for attempting a preliminary discussion. The theoretical part of the present paper has been held back for over a year, the first part of which was spent in an unsuccessful effort to find some less revolutionary interpretation of the observed motions than that here tentatively put forward, the second part being spent in an effort, again unsuccessful, to connect up the suggested interpretation with the general scheme of the theory of relativity. The paper is now put forward, but with all possible reserve, partly in the hope of provoking discussion, but partly with the humbler aim of drawing attention to the importance and difficulty of the problem.

* *Astrophys. Journ.*, 57 (1923), 274.

The mathematical analysis in the paper is not of any special difficulty, but the physical conceptions are somewhat unusual. I have therefore attempted to give a simple explanation of these, freed from all mathematical analysis, in the summary at the end of the paper (§ 16). The majority of readers may prefer to turn at once to this.

Analysis of the Observed Nebular Motions.

2. The measurements of v. Pahlen * have established that the arms of normal spiral nebulæ are approximately equiangular spirals, at any rate over a large part of their length. The movements which are in progress in these arms may change the shapes of the curves, but it is very improbable that they should change them to anything except new equiangular spirals. For if we supposed the equiangular-spiral shape to represent a merely transitory phase in nebular development, we should also have to suppose that all spirals passed through this phase at just such instants that the light from them all happened, by a marvellous coincidence, to reach our system at just the same moment—the present.

The condition that the internal movements can only change the equiangular-spiral arms into new equiangular spirals provides a check on the observed measurements of internal motions, and further enables us to embrace a large number of measured motions in a single simple formula with the assurance that, in so far as the measurements do not fit the formula, it is the former, and not the latter, that stand in need of adjustment.

3. Suppose that a spiral arm has for its equation at the present moment

$$r = \mathrm{A}e^{\theta \tan \alpha} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where α is the angle of the spiral, which is constant throughout its length. Differentiating this equation with respect to the time (logarithmically) we find

$$\frac{1}{r}\frac{dr}{dt} = \frac{1}{\mathrm{A}}\frac{d\mathrm{A}}{dt} + \frac{d\theta}{dt}\tan\alpha + \theta\frac{d\alpha}{dt}\sec^2\alpha \quad . \quad . \quad . \quad (2)$$

in which A and α have been supposed subject to differentiation because the spiral is supposed to be itself changing its constants, although not losing its spiral form. From this equation we have at once

$$\frac{dr}{dt}\cos\alpha - r\frac{d\theta}{dt}\sin\alpha = r\left[\frac{1}{\mathrm{A}}\frac{d\mathrm{A}}{dt}\cos\alpha + \operatorname{cosec}\alpha\log\left(\frac{r}{\mathrm{A}}\right)\frac{d\alpha}{dt}\right] \quad . \quad (3)$$

Van Maanen resolves his measured motions into two rectangular components, μ_{stream} along the nebular arms, and μ_{trans} at right angles to the arms. Clearly μ_{trans} is equal to the quantity which stands on the left-hand side of equation (3). Hence, if the observed motions are to transform the present equiangular spiral into a new equiangular spiral, we must have

$$\mu_{\text{trans}} = r(a + b\log r) \quad . \quad . \quad . \quad . \quad (4)$$

* *Ast. Nach.*, 4503 (1911).

where a and b are constants. This relation is necessary and sufficient to ensure that the new curves shall be equiangular spirals.

4. In the nebula which he has measured last and most fully, namely M 33,* Van Maanen has tabulated μ_{trans} for six values of r. As the following table shows, the values of μ_{trans} fit a formula of the type $r(a+b\log r)$ exceedingly well.

r.	No. of Points.	μ_{trans} (obs.).	$r(a+b\log r)$ (calc.).
′		″	″
2·25	26	+0·002	+0·002
4·5	100	+0·001	+0·001
7·5	103	−0·003	−0·002
10·5	98	−0·004	−0·005
13·5	69	−0·009	−0·009
Just over 15	3	−0·019	−0·016

In two other nebulæ, M 63 † and N.G.C. 2403,‡ the fit is less good, although probably well within the limits of error. The following table shows the best fit for M 63 :—

r.	No.	μ_{trans} (obs.).	$r(a+b\log r)$ (calc.).
′		″	″
0·75	11	+0·004	+0·004
1·5	36	+0·005	+0·005
2·5	32	+0·002	+0·006
3·5	12	+0·006	+0·006
4·5	3	+0·015	+0·006

The best fit for N.G.C. 2403 is as follows :—

r.	No.	μ_{trans} (obs.).	$r(a+b\log r)$ (calc.).
′		″	″
1·4	23	+0·009	+0·007
3·1	37	+0·004	+0·009
5·1	11	+0·012	+0·009
7·3	3	−0·001	+0·008
8·9	2	+0·010	+0·006

The only other nebulæ for which μ_{trans} is given for different values of r is M 94 = N.G.C. 4736, but as it is tabulated for only two values, it is naturally possible to make the formula $(a+b\log r)$ fit the observations. We can, however, use the two given values of r to determine the values of a and b.

5. Collecting results it appears that the observations fit the formula $r(a+b\log r)$ to within errors of observation in every nebula in which a comparison can be made, a possible exception arising in the outermost parts of the nebulæ, in which the equiangular spiral form is not, in any

* *Ap. Journ.*, 57 (1923), 274. † *Ibid.*, 56. ‡ *Ibid.*, 56 (1922), 207.

case, very closely adhered to. Further, the calculated values of a and b have the same signs in all the cases we have considered, a being uniformly positive and b being uniformly negative. It tends to simplicity to write

$$a + b \log r = b \log \left(\frac{r}{r_0}\right),$$

and it then appears that the positive value of a merely indicates that r_0 is in every case greater than one minute of arc. By comparison with equation (3)

$$b = \operatorname{cosec} a \frac{da}{dt},$$

so that the uniformly negative values for b which have been obtained denote that a is decreasing for each nebula—*e.g.* all the nebulæ are curling up more tightly than at present.

For M 33, the nebula for which it is possible to determine b with most precision, we have $a = 39^\circ\ 39'$, and $\sin a = 0{\cdot}638$. The table on p. 62 was calculated for $b = -{\cdot}000014$, the unit for r being $1'$ of arc, the unit of time a year, and the logarithms being to base e. Thus

$$\frac{da}{dt} = b \sin a = -{\cdot}000009\ (\text{year})^{-1},$$

showing that a is decreasing at the rate of about 1° in 2000 years.

For the other nebulæ the values of b are very much less reliable, but they appear to be about $b = -{\cdot}00003$ for M 63, and $b = -{\cdot}000016$ for M 94. The value of a appears to be about 6° for M 63 and perhaps 10° for M 94. From these figures it follows that in each of these nebulæ a is decreasing at about 1° per 6000 years.

These figures are at best only approximate, but the circumstance that the signs of all the values of b agree suggests that there is something real in them. If they can be trusted, it would appear that the spiral nebulæ are far from being in a steady state, and that the spiral form must be of comparatively short duration. This would fit in with the fact to which Reynolds has recently drawn attention, that only a small fraction of the total number of nebulæ show the spiral form at all. It would be of interest to examine whether any correlation could be traced between the angles of spirals and their apparent stages of development in the sense of the less developed spirals showing the larger angles.

Analysis of Nebular Forces.

6. We pass to the main problem, and attempt to discover what forces can produce the observed motion.

If f_x, f_y, f_z are the components of acceleration in three rectangular directions in space, the equations of motion are

$$f_x = \mathrm{X} - \frac{1}{\rho}\left(\frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{xy}}{\partial y} + \frac{\partial p_{xz}}{\partial z}\right) \quad . \quad . \quad . \quad (5)$$

If we disregard the non-symmetry of pressure arising from the action of viscosity, etc., this reduces to the more familiar equation

$$f_x = \mathrm{X} - \frac{1}{\rho}\frac{\partial p}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

These equations are quite general, in the sense of taking into account the action of all forces, both known and unknown, provided we agree that the force (X, Y, Z) is to include forces of all kinds which act from a distance, and that p_{xx}, etc., are to include all forces arising from the action of contiguous particles, whether of matter or of "ether." Electromagnetic forces, including pressure of radiation, may be reckoned in either category, but must not be reckoned in both. In point of fact they may be omitted altogether, for a brief calculation shows that they are totally inadequate to produce any appreciable fraction of the observed accelerations. The known forces accordingly reduce to gravitation and the action of gas-pressure, the latter including viscosity and all effects of the motion, regular and irregular, of the molecules.

If we integrate equation (6) along the path of a particle, from point 1 to point 2, we obtain

$$\tfrac{1}{2}(v_2^2 - v_1^2) = \int_1^2 (\mathrm{X}dx + \mathrm{Y}dy + \mathrm{Z}dz) - (q_2 - q_1) \quad . \quad . \quad (7)$$

where v_2, v_1 are the velocities of the points 1, 2 and

$$q = \int \frac{dp}{\rho}.$$

The precise form to be assigned to q will of course depend on the variations of temperature experienced by the particle in its motion. If we assume the gas to obey the adiabatic law throughout, the ratio of the specific heats being γ, we readily find that

$$q_2 - q_1 = \frac{\gamma}{3(\gamma - 1)}(\mathrm{C}_2^2 - \mathrm{C}_1^2),$$

where C_1, C_2 are the molecular velocities (mean-square) at the points 1, 2. We know that C_1, C_2 are at most of the order of 1 km. a second.

To get some idea of the order of magnitude of v_1 and v_2 let us consider the special case of the nebula M 33. Its distance has been determined as 2000 parsecs, so that an observed motion of 1″ per annum means a linear velocity of 10,000 kms. a second. Thus Van Maanen's measurements represent linear velocities which vary from 130 kms. a second in the inner regions of the nebula ($r = < 3'$) to 280 kms. a second in the outermost regions ($r = 15'$). The velocities in this particular nebula are not unusually large, so that in general we may suppose that v_1 and v_2 are of the order of 40,000 (km. per sec.)^2.

We have seen that the utmost amount that the term $q_2 - q_1$ in equation (6) can contribute to the value of $v_2^2 - v_1^2$ is of the order of one (km. per sec.)^2, so that any acceleration produced by gas-pressure may be treated as entirely negligible. The argument is valid regardless of

the particular relation assumed to hold between pressure, density, and temperature in the gas, and can readily be extended so as to apply to the most general types of pressure contemplated in equation (4). Indeed this general conclusion becomes obvious when the problem is viewed from the standpoint of conservation of energy. Acceleration under gas-pressure results from the kinetic energy of the gas molecules being transformed into body-motion of the mass; in the particular problem we have been considering, the surrender by the molecules of the whole of their kinetic energy of molecular motion could only change the kinetic energy of the mass about one forty-thousandth part of its value. Visual evidence of the smallness of the effect of gas-pressure is provided by the closeness with which the arms of the spirals are confined to one plane; if gas-pressure were effective in comparison with the other agencies at work, it would rapidly blow the spiral arms out into three-dimensional structures.

Unless we suppose the universe to consist of something more than matter and ether, the forces transmitted by the action of contiguous particles can consist only of electromagnetic forces (transmitted through the ether) and mechanical pressure (transmitted by matter). These have both been seen to produce quite inappreciable effects, so that equations (5) and (6) may be replaced by

$$f_x = \mathrm{X}.$$

We conclude that the accelerations in the nebular arms are produced by action from a distance; from now on it will be immaterial whether we speak of the acceleration of a particle or of the force per unit mass acting upon it. The only known force, apart from electromagnetic forces already dismissed, which acts upon matter from a distance is gravitation. Let us, then, examine whether gravitation is adequate to account for the motions observed in the nebular arms.

Orbits under Gravitation.

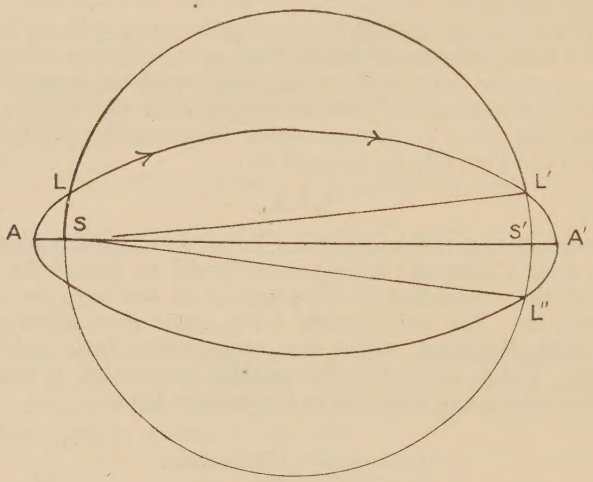
7. Van Maanen's measurements have shown that as a general rule * the linear velocity increases on passing outwards in a nebula; the rate of description of areas is also found to increase. It follows that if the particles in the spiral arms of a nebula are describing orbits merely in the gravitational field of the nebula itself, the outermost particles, which started first from the nucleus, must have been endowed both with more energy and with more angular momentum than the later particles. If the various particles are describing elliptic orbits, it follows that the eccentricities, major axes, and periodic times must all be greater for the earlier particles than for the later ones.

Thus if the various particles are acted upon merely by gravitation, their orbits must differ widely one from another. We have to consider whether it is possible for particles describing these various elliptic orbits so to move that their configurations and motions agree with what is observed.

* Of the nebulæ so far measured, M 81 forms the only exception, and this only if the early measurements (*Ap. Journ.*, **44** (1916), 210) can be entirely relied upon.

The outermost particles in a nebula are in general at a distance from the nucleus equal to anything from five to twenty (at least) times the radius of the nucleus. Assuming that they originally came out of the nucleus, their orbits must be of quite high eccentricity, and so must be comparatively flat; if they are ellipses they must approximate to parabolas, while if they are hyperbolas they must approximate to straight lines. In either case the velocity of the particle which is describing the orbit ought to be mainly radial.

This, however, is precisely the opposite of what is revealed by Van Maanen's measurements. In not a single nebula do radial velocities predominate, and in most the radial velocity is less than half of the rotational velocity. This consideration appears to dispose of the possibility that the orbits are being described under gravity, but as the



matter is one of some importance, it will be well to consider it somewhat more fully in its numerical aspects.

In the figure, ALL′A′ represents an elliptic orbit of high eccentricity about the focus S, the eccentricity of this particular ellipse being 0·9. A circle drawn on the focal line SS′ as diameter cuts this in the points LL′, so that the angles SLS′ and SL′S′ are each right angles. Thus at each of the points LL′, the orbit is inclined at an angle of 45° to the focal radii SL, SL′. Throughout the range LL′, the angle is less than 45°, reaching a minimum value of $\cos^{-1}e$ at the end of the minor axis. As a particle describes this orbit under the gravitational attraction of a mass at S, the time during which the inclination of its direction of motion to the focal radius is less than 45° is represented by the time taken to describe the arc LL′, and so by the area LSL′, while the time during which this inclination is greater than 45° is represented by the time taken to describe the arcs AL, L′A′, and so by the sums of the areas ASL and A′SL′. So long as we are dealing with ellipses of high

eccentricity, SL and S'L' may be supposed without appreciable error to be perpendicular to the major-axis AA', whence we find

$$\begin{aligned} \text{Area ASL} &= \tfrac{1}{2}b^2(1-e) \\ \text{,, LSL'} &= \tfrac{1}{2}\pi ab - b^2 \\ \text{,, L'SA'} &= \tfrac{1}{2}b^2(1+e). \end{aligned}$$

It follows that as the particle describes its orbit the inclination of its direction of motion to the focal radius is greater than 45° for a fraction $2(1-e^2)^{\frac{1}{2}}/\pi$ of the whole time of the orbit. When $e=0{\cdot}9$ the value of this fraction is 0·31. A more exact calculation shows that it is equal to one-half when $e=0{\cdot}73$. If the particles in nebular arms are describing ellipses under gravitation, all particles which are at more than six radii of the nucleus away from the nucleus must be describing ellipses of eccentricity greater than 0·73. Hence we should expect that at all distances greater than about six radii of the nucleus away from the nucleus, the inclination of the direction of motion to the focal radius would be less than 45°, or, what is the same thing, that the radial component of velocity would be greater than the rotational. We have already noticed that precisely the reverse is the case; to how great an extent is shown in the following table, which gives observed ratios of radial to rotational velocities in various nebulæ. The entries to which references are attached are given directly by Van Maanen; the others have been calculated from his published measurements.

Nebula.	Ratio of Radial to Rotational Velocity.	
	Whole Nebula.	Outer Regions only.
M 101	0·32*	...
M 33	0·48*	0·13§
M 51	0·42*	...
M 81	0·34*	...
M 63	0·32	0·34
N.G.C. 2403	0·93†	1·16
M 94	0·50‡	0·68‡

The case against elliptic orbits described under gravitation is in effect much stronger than appears in the table, for throughout the whole range of all the arms of all the nebulæ for which the velocities have been measured, not a single region has been found in which the radial velocity systematically exceeds the rotational velocity, with the single exception of the outermost region ($r>5'{\cdot}1$) of the nebula N.G.C. 2403. We could only maintain the hypothesis of elliptic orbits by supposing that the vast majority of the moving particles happened to be quite close to the aphelia of their orbits—a supposition which would involve a violent straining of the laws of probability. On the evidence of the present measurements we can only conclude that we are not free to suppose that the motions take place under gravitation, and that we must look to some new and so far unknown force for an explanation.

* *Astrophys. Journ.*, 54 (1921), 356.
† *Ibid.*, 56 (1922), 207.
‡ *Ibid.*, 56 (1922), 212.
§ *Ibid.*, 57 (1923), 273.

Orbits under Generalised Force.

8. Throughout the greater part of the length of the nebular arms the motion is, to a first approximation, along these arms. Thus we may suppose, still only to a first approximation, that each particle describes an equiangular spiral in space, following exactly in the track of its predecessor. This suggests (although it does not quite prove) that to a first approximation the motion may be regarded as motion in an unchanging field of force, motion of the type described hydrodynamically as "steady motion."

The motion cannot, of course, be entirely "steady"; for the continual passage of matter from the nucleus to the arms must produce some change in the field of force, but this change will be of the type usually designated as "secular," and so may be disregarded in working only to a first approximation. The disregard of these secular changes is probably equivalent to a disregard of the quantities a and b which we considered in §§ 3–5.

9. We now proceed to investigate what types of forces are capable of producing steady motion of the type just described—*i.e.* a steady motion in which each particle moves along the spiral arms of the nebula.

Let the equation of the spiral curve in plane polar co-ordinates be assumed as before to be

$$r = Ae^{\theta \tan \alpha}$$

For the motion of a particle moving along this curve, we obtain by logarithmic differentiation,

$$\dot{r} = r\dot{\theta} \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Let F, G denote the accelerations of the particle (or the resultant force per unit mass) in the directions of r and θ increasing. Using equation (8) we have

$$F = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - \frac{\dot{r}^2}{r} \cot^2 \alpha \quad . \quad . \quad . \quad . \quad (9)$$

$$G = \frac{1}{r} \frac{d}{dt}(r^2\dot{\theta}) = \left(\ddot{r} + \frac{\dot{r}^2}{r}\right) \cot \alpha \quad . \quad . \quad . \quad (10)$$

whence solving for $\ddot{r}$ and $\dot{r}^2$,

$$\ddot{r} = (G \cot \alpha + F) \sin^2 \alpha \quad . \quad . \quad . \quad . \quad (11)$$

$$\dot{r}^2 = r(G \tan \alpha - F) \sin^2 \alpha \quad . \quad . \quad . \quad . \quad (12)$$

The resultant velocity v is given by $v = \dot{r} \operatorname{cosec} \alpha$, so that from equation (12)

$$v^2 = r(G \tan \alpha - F) \quad . \quad . \quad . \quad . \quad (13)$$

an equation which could have been obtained directly by writing down the acceleration of the particle normal to its path.

Since $\frac{d}{dr}(\dot{r}^2) = 2\ddot{r}$, we obtain at once from equations (11) and (12)

$$\frac{d}{dr}[r(G \tan \alpha - F)] = 2(G \cot \alpha + F),$$

leading to

$$3F + r\frac{\partial F}{\partial r} = G(\tan\alpha - 2\cot\alpha) + r\frac{\partial G}{\partial r}\tan\alpha \quad . \quad . \quad (14)$$

This is the necessary relation between F and G, the two components of acceleration (or of force per unit mass) when a particle describes an equiangular spiral under a steady field of force. The observations of Van Maanen leave little room for doubt that this equation must be satisfied, at least to a fair approximation, throughout the greater part of the path of a particle in the spiral arms of a nebula.

The equation being of the first degree in F and G, and of zero dimensions in r throughout, it is clear that F and G must be proportional to the same power of r, or of course to sums of powers such that the same powers enter into each sum. If v can be determined from observation as a function of r, the values of F and G follow at once from equations (13) and (14).

10. The nebula for which v has been tabulated most fully as a function of r is M 33, for which v is found to vary approximately as r^n where $n = 0{\cdot}26$. This is shown in the following table:—

r.	No.	v (obs.).	$Cr^{0\cdot26}$.
′		″	″
2·25	26	0·013	0·014
4·5	100	0·017	0·017
7·5	103	0·022	0·020
10·5	98	0·023	0·022
13·5	69	0·023	0·023
Just over 15	3	0·021	0·024

For N.G.C. 2403 the best fit is again obtained by taking $n = 0{\cdot}26$, the result being as shown below:—

r.	No.	v (obs.).	$Cr^{0\cdot26}$.
′		″	″
1·4	23	0·015	0·015
3·1	37	0·019	0·019
5·1	11	0·021	0·021
7·3	3	0·028	0·023
8·9	2	0·040	0·025

For M 63 the solution for a best fit yields $n = 0{\cdot}21$, but an agreement well within the limits of error of measurement is again obtained by taking $n = 0{\cdot}26$. Both sets of values are shown in the following table:—

r.	No.	v (obs.).	$Cr^{0\cdot21}$.	$C'r^{0\cdot26}$.
′		″	″	″
0·75	11	0·016	0·016	0·015
1·5	36	0·018	0·018	0·018
2·5	32	0·020	0·020	0·020
3·5	12	0·022	0·022	0·022
4·5	3	0·021	0·024	0·024

For N.G.C. 4736 * Van Maanen evaluates v (or μ_{stream}) for only two values of r, and these can be made to fit a formula of the type Cr^n as far as the last figure given for any value of n between 0·18 and 0·27.

The only other measurements which yield a value for n are the early measurements on M 101.† A solution for closest fit gives $n = -0{\cdot}40$, and it is doubtful whether a solution with $n = 0{\cdot}26$ would be within errors of observation. The two solutions are shown in the following table:—

r.	No.	μ_{rot}.	v.	$Cr^{-0\cdot40}$.	$C'r^{0\cdot26}$.
2·2	14	0″·026	0″·027	0″·029	0″·018
3·9	16	0·024	0·025	0·023	0·021
5·9	15	0·015	0·016	0·019	0·023
8·9	17	0·021	0·022	0·016	0·025

Except for the last nebula, the variation of velocity with distance is well represented by the supposition that $n = 0{\cdot}25$, so that v^2 varies as $r^{\frac{1}{2}}$ and equation (13) now shows that F and G must each vary as $r^{-\frac{1}{2}}$.

11. Turning back to equation (14), it is natural to examine first whether the orbits can be described under central forces only. Putting $G = 0$, equation (14) becomes

$$3F + r\frac{\partial F}{\partial r} = 0,$$

showing that F would have to vary as r^{-3}. In fact the equiangular spiral is a special case of Cotes' spiral which, as is well known, is the orbit described under a central force varying inversely as the cube of the distance. But equation (13) shows that under such a force v would vary inversely as the distance ($n = -1$), a law which it is quite impossible to reconcile with Van Maanen's measurements.

12. Let us next examine the possibility of the motion taking place under a purely tangential force. Putting $F = 0$, equation (14) becomes

$$(1 - 2\cot^2\alpha)G + r\frac{\partial G}{\partial r} = 0,$$

so that G must vary as r^p, and hence v must vary as $r^{\frac{1}{2}(p+1)}$ where

$$p = 2\cot^2\alpha - 1.$$

For all nebulæ for which measurements have been made, α is less than 45°, so that p is greater than unity. Thus the velocity would have to increase more rapidly than r as we receded from the centre ($n > 1$), and this again is quite impossible to reconcile with the measurements.

13. Thus the observed nebular motions indicate that both radial and tangential forces must be at work; F and G must both be different from zero. We already know that F must contain a term $-M/r^2$

* *Ap. Journ.*, 56 (1922), 212.

† *Ibid.*, 44 (1916), 223. In this nebula the average radial velocity is 0·32 times the rotational, so that $v = 1{\cdot}05\ \mu_{\text{rot}}$.

originating in the gravitational attraction of the nucleus. If this represented the whole value of F, the solution of equation (14) would be

$$G = \frac{M}{r^2(\tan\alpha + 2\cot\alpha)} + Cr^p,$$

where again p stands for $2\cot^2\alpha - 1$. Since p is greater than unity, C must vanish; otherwise we should be postulating forces which increased beyond limit as we receded from the central nucleus. Putting $C = 0$ in the value just obtained for G, equation (13) gives

$$v^2 = \frac{M}{r(1 - \frac{1}{2}\sin^2\alpha)},$$

so that v would have to vary as $r^{-\frac{1}{2}}$. This again cannot be reconciled with the measurements, so that we conclude the existence of both radial and tangential forces in addition to the gravitational force arising from the central nucleus.

14. Assuming that v varies as r^n, equation (13) shows that F and G each vary as r^{2n-1}. Thus in equation (14) we may write

$$r\frac{\partial F}{\partial r} = (2n-1)F, \quad r\frac{\partial G}{\partial r} = (2n-1)G,$$

and the equation assumes the form:—

$$F = G\,\frac{n\tan\alpha - \cot\alpha}{1+n}$$

or

$$F = -G\cot(\alpha+\beta) \quad . \quad . \quad . \quad . \quad (15)$$

where β is given by $\tan\beta = n\tan\alpha$. We have already seen that n is a small quantity (about 0·25), and since $\tan\alpha$ is fairly small, it follows that β is a comparatively small angle. Taking $n = 0{\cdot}25$, the value of β is found to be 3° 20′ for M 51, and 10° 43′ and 11° 18′ respectively for the two arms of M 33. If β could be neglected altogether, equation (15) would reduce to $F = -G\cot\alpha$, so that the acceleration, and so also the resultant force, would be perpendicular to the motion, its direction being inwards. In actual fact we see that the resultant force is inclined at the small angle β to the normal to the path.

It is clear that no stress ought to be laid on the particular values of β just calculated, for they have proved to be of the degree of smallness of various small quantities which have been neglected altogether. But the circumstance that, in most nebulæ, the velocity steadily increases as we pass outwards from the centre suggests (although it does not rigorously prove) that β is in actual fact different from zero, and that it has the sign we have attributed to it. For if β were zero, the force (F, G) would do no work on the moving particle, so that its velocity would remain constant throughout its path. The differences of velocity observed in different parts of the nebula would then have to be attributed to differences in the velocity of projection of the particles from the nucleus.

The point is of some consequence as providing further evidence, in

addition to that already given in § 7, against the possibility of the force (F, G) arising merely from the ordinary gravitational field of nucleus plus arms. In such a field the force from the nucleus would be directed towards the centre, while that from the arms would be in a direction intermediate between this and that of the inward normal. The sign of β would accordingly be opposite to what we have found, except in the single instance of M 81.

15. Although the possibility just considered still cannot be entirely dismissed, it seems more natural, on the present evidence, to conjecture that the orbits we have been considering are described under the action of some sort of generalised gravitational force which falls off approximately as $r^{-\frac{1}{2}}$ and is not directed towards the nucleus. It cannot be claimed that it has been shown that such a force exists, but the assumed existence of such a force appears to provide the simplest (and, so far as the present writer has been able to discover, the only reasonable) explanation of the motions observed by Van Maanen in the arms of the spiral nebulæ. A force falling off as $r^{-\frac{1}{2}}$ might, of course, easily be insignificant in comparison with ordinary gravitation over such distances as occur within the limits of the solar system and yet preponderate entirely over it at distance of the order which occur in the spiral nebulæ.

A difficulty arises if we regard the hypothetical new force as arising simply from an addition to the present gravitational field, so that the forces are functions merely of position in space. For the orbits of particles moving under a fixed field of force form a doubly infinite system, whereas the equiangular spirals having the nucleus as origin form only a singly infinite system. Indeed equation (13), namely

$$v^2 = r(\mathrm{G}\tan\alpha - \mathrm{F})$$

expresses the condition that a particle shall describe a spiral of angle α, and unless the velocity of projection v and the angle of projection α conform to this equation at the outset, the particle cannot describe an equiangular spiral at all. The difficulty is not insurmountable; it can, for instance, be avoided by the supposition that equiangular spirals are merely asymptotic curves to which a doubly infinite family of orbits approach for large values of r. It will nevertheless be found to restrict the possible interpretations of the hypothetical new force.

A more serious difficulty to regarding the new force as arising from a permanent field is that the direction of the force appears to depend on the direction of the motion of the particle upon which it acts. This suggests that the specification of the force must in some way depend on the motion as well as the position of the particle it affects, as is, for instance, the case with electromagnetic forces.

It may be of interest to mention two instances of forces which are not open to either of the objections just mentioned.

I. A central force defined by $\mathrm{F} = -\frac{v^2}{r}$, $\mathrm{G} = 0$. The orbits form a singly infinite family. All are equiangular spirals, and v varies as $1/r$.

II. A force kv^2/r, where $k < 1$, acting in a direction at right angles

to the velocity v of the particle. The orbits form a doubly infinite family, but all approximate when r is large to equiangular spirals of angle $\cos^{-1} k$, and v remains constant throughout the description of any single orbit.

Obviously neither of these illustrations can represent the motions actually observed in spiral nebulæ, although possibly each may lead one step in the right direction.

Summary and Conclusion.

16. It may be convenient, in summarising the argument which has been set out, to use a simple analogy.

Imagine that the filaments of the spiral nebulæ all over the sky, instead of being equiangular spirals, were observed to be concentric circles surrounding the nuclei. An observer, measuring the motions of condensations in these filaments might resolve each measured motion into two components, one radial and one tangential. It is, however, clear that unless all the measured radial velocities on any one circle were precisely equal, the measures would indicate that the circle was losing its circular shape, and this would be contrary to the observed uniformity of the whole sky. It would be permissible to assume that the predicted loss of circular shape arose only from inaccuracies of measurement, and to suppose that each radial velocity on any circle was equal to the average of all the radial velocities measured on that circle. If this average proved to be zero, we might legitimately conclude that the circle was neither expanding nor contracting, and that the particles simply followed one another round in circular orbits. (An argument on these lines has led to the conclusion that to a first approximation the particles in the normal spiral nebula simply follow one another along the spiral arms, except perhaps in the extreme outermost and innermost parts of the nebula.)

Suppose, now, that our imaginary observer measured the linear velocity v in each circle of a nebula which consisted of a large number of concentric circles. By calculating the value of v^2/r for each circle he could estimate the intensity of the force which retained the moving particles in their circular orbits. If the calculated values of v^2/r proved to be proportional to $1/r^2$, he would infer that the forces in question were the gravitational forces arising from the nucleus, and he could calculate the mass of this nucleus. But if the values of v^2/r did not prove to be even approximately proportional to $1/r^2$, he would be forced to conclude that the moving matter was retained in its orbits by some force other than the ordinary gravitational force of the nucleus.

Our argument and procedure have been quite similar to that of our imaginary observer. A spiral is a more complicated curve than a circle, so that our discussion has of necessity been framed in mathematical terms, but has been essentially similar to that just given. And it has been found that the restraining force does not fall off as $1/r^2$, but as $r^{-\frac{1}{2}}$; further, it is not directed towards the nucleus, but its direction depends on the direction of motion of the particle on which it acts, being nearly perpendicular to that direction.

In the case of a nebula of approximately zero angle—*i.e.* a nebula in which the constant angle between the tangents to the nebular arm and a circle surrounding the nucleus, is very small—our argument becomes identical with that of our imaginary observer just considered. In one of the nebulæ we have discussed, M 63, the angle is in point of fact only about 6°, and the orbits do not differ widely from circles surrounding the nucleus. In this nebula v^2/r is found to be approximately proportional to $r^{-\frac{1}{2}}$, and is not in the least degree proportional to $1/r^2$. In N.G.C. 4736, of only slightly greater angle, similar conditions prevail.

Reasons have been found for supposing that ordinary gravitation, gas-pressure, viscosity, pressure of radiation, and electromagnetic forces are all incapable of producing the type of motion observed. In particular, they are incapable of accounting for the great observed increase of velocity and the still greater increase of angular momentum as the distance from the nucleus is increased, and the only course open seems to be to assume that under nebular conditions the gravitational field is somehow modified in such a way that conservation of energy and of angular momentum both disappear.

The general effect of the modified gravitational force appears to be one of seizing the particle and compelling it to revolve about the nucleus of the nebula with an angular velocity which approximates to that of the nucleus itself rather than to that demanded by the conservation of angular momentum. As a consequence, the particle moves as though it were repelled from the nucleus, and describes an orbit of ever-increasing radius with ever-increasing velocity. We can see that this second effect is merely a consequence of the first by considering the limiting case in which the angular velocity of the particle is kept precisely equal to that of the nucleus. Suppose that in ordinary polar co-ordinates the angular velocity, instead of being determined by the condition for conservation of angular momentum, namely

$$r^2\dot{\theta} = \text{constant},$$

is defined by

$$\dot{\theta} = \omega,$$

where ω is the angular velocity of the nucleus, a constant. Neglecting the gravitational attraction of the nucleus altogether, the equation of radial motion is now found to be

$$\ddot{r} - r\dot{\theta}^2 = 0$$

or

$$\ddot{r} = r\omega^2,$$

from which the integral equation of the orbit is found to be

$$r = Ae^{\theta} + Be^{-\theta}.$$

For large values of r this approximates closely to the equiangular spiral $r = Ae^{\theta}$ of angle 45°. The velocity of course increases directly as r.

We have not so far succeeded in obtaining a satisfactory analytical specification of the imagined new force. Its direction appears to depend on the direction of motion of the particle on which it acts, so that it is

likely that its specification may depend on the particles velocity. We have conjectured that the force ought perhaps to be considered, in part at least, to be proportional to v^2/r rather than to $r^{-\frac{1}{2}}$. In actual nebulæ the two specifications amount to the same thing, since v^2 is found to vary approximately as $r^{\frac{1}{2}}$.

17. It is quite clear that a force of the kind we have been considering, if it exists at all, can be of only insignificant magnitude in the solar system, or at least can differ only very slightly from the ordinary gravitational force. The conjectured new force may be regarded either as something which has to be added to ordinary gravitation, or as something into which gravitation transforms when the masses and distances are on a colossal scale; the alternatives of course represent different points of view, and not different possibilities of fact. We may notice that in the case of a planet describing a circular orbit, the gravitational force may equally well be described either as v^2/r, or as M/r^2, or as a mixture of the two in any proportions we please; it is only for orbits of appreciable eccentricity that the effects of these forces diverge. It is at least conceivable that a slight step from a gravitational force M/r^2 in the direction of a force v^2/r might be found to provide explanations of some of the unsolved enigmas of the solar system, such as the moon's motion in longitude, Bode's law, and the circularity of planetary orbits.

Another phenomenon in which the conjectured new force might possibly play a part is the separation of the two components of a binary star after fission. The laws of conservation of energy and momentum restrict very severely the extent to which the two constituents can separate, and there is a serious difficulty in reconciling these restrictions with the observed present distances of the components of binary stars. Under the influence of the conjectured new gravitational force the constituents might be supposed to describe orbits of a spiral nature about one another, their distances apart continually increasing.

A very real difficulty is removed by supposing that the conjectured new force is in operation in the nucleus of a spiral nebula as well as in the arms. Helmholtz calculated that the time necessary for gas-viscosity to reduce velocity differences to one-half in an atmosphere of 8 kilometres depth is of the order of 40,000 years, and Poincaré* has drawn attention to the general inability of gas-viscosity to secure approximately uniform rotation in a nebula within a reasonable period of time. In general the time required to reduce velocity differences to one-half is of the order of $R^2\rho/\gamma$, where R is the radius of the nebula and γ its coefficient of viscosity. Remembering that γ is independent of the density in a gas, and does not vary rapidly with the temperature, let us put $\gamma = 0{\cdot}0002$, this being the value of the coefficient of viscosity for ordinary air. Then the time for a nebular nucleus 2 light-years in diameter and of density $\rho = 10^{-17}$ is found to be of the order of 10^{14} years! Yet spectroscopic measurements show that the nuclei of nebulæ such as N.G.C. 4594 (in Virgo) and M 31 (the Andromeda nebula) are rotating with approximately uniform angular velocity. Our hypothetical new force might be found to provide a much needed explanation of this uniform rotation.

* *Leçons sur les Hypothèses Cosmogoniques*, p. 29.

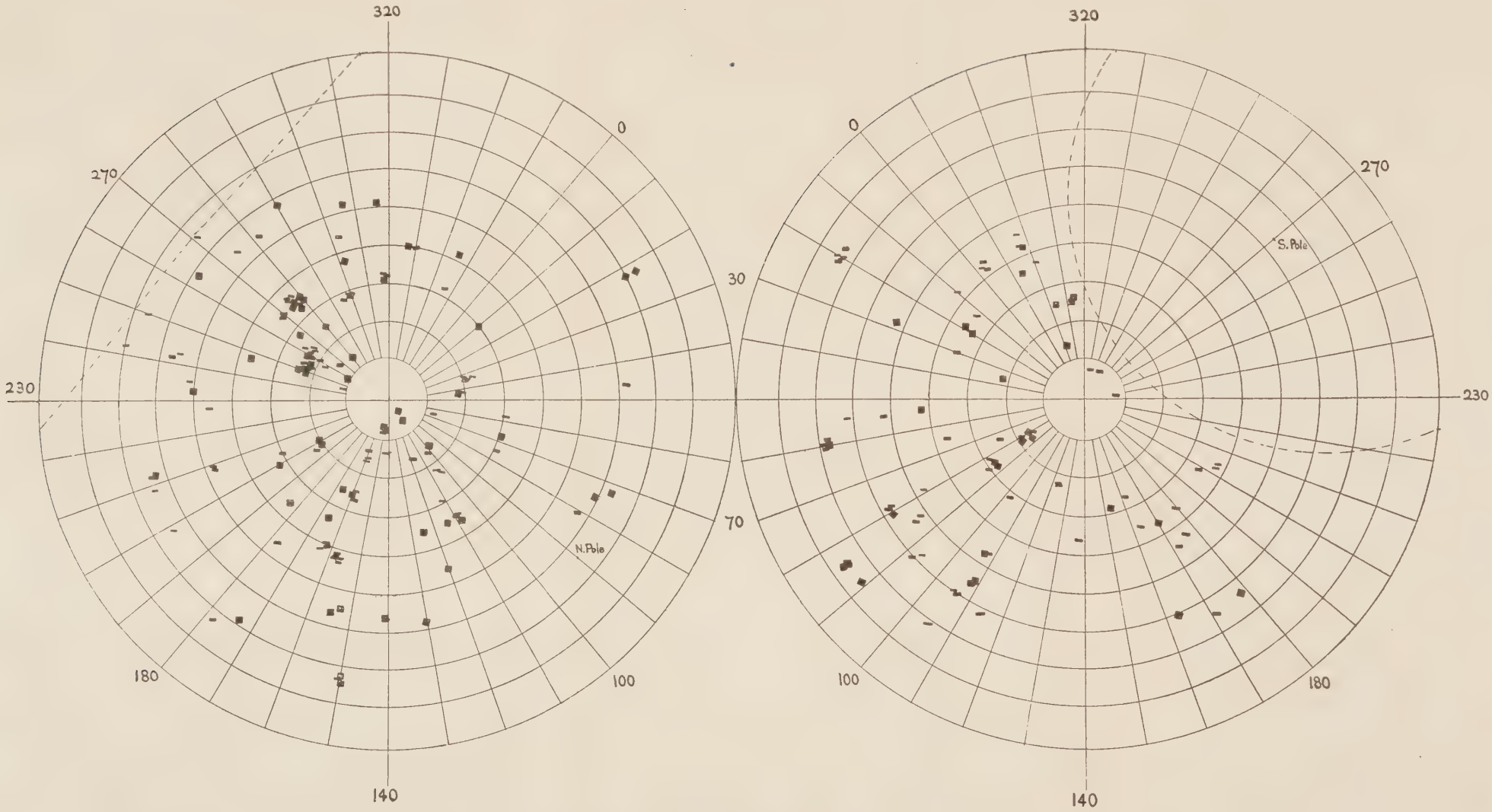
Finally the force might be expected to exhibit itself on the grandest scale of all in the motions of bodies under the gravitational field of the whole universe. Arguing from the analogy of the nebular arms, we should expect the outermost parts of the universe to be receding into space along spiral orbits. Possibly we may find here an explanation of the velocities of recession of the spiral nebulæ.

18. It will of course be understood that there is nothing final in the conclusions of the present paper, and that further measurements on nebular motions may compel their modification or abandonment. The simplest conceivable interpretation of the observed motions, namely that they are motions in the ordinary gravitational field of the nucleus, has been rejected, not because the observed motions would conflict with the laws of dynamics, but because they would conflict with the laws of probability. The argument against the field being of the Newtonian type is a purely statistical argument, and, as such, is liable to be overthrown by the accumulation of more material.

At the same time the chance of overthrow would appear to be very slight. The supposition of an ordinary Newtonian field would require that almost all of the points for which measurements have so far been taken should happen to lie simultaneously and now on the small arcs of their orbits represented by L′A′ in fig. 1. It further requires us to suppose that the almost universal spiral form is of the nature of a coincidence. Under a Newtonian field of force an equiangular spiral would rapidly change into something different from an equiangular spiral. But although it would be straining the laws of probability very badly to advocate a purely Newtonian field on the present evidence as to the internal motion of nebulæ, the possibility is one which cannot be entirely ruled out.

The Galactic Distribution of the Small Spiral Spheroidal and Lenticular Nebulæ. By J. H. Reynolds. (Plates 1, 2.)

In previous papers the writer has dealt with the galactic distribution of the spiral nebulæ of 2′ diameter and upwards. The present paper, which must be regarded as preliminary only, deals with the nebulæ under 2′ diameter of the spiral class, but not necessarily spiral in structure. These nebulæ are very numerous, and the smaller and fainter examples must number many thousands. No attempt has been made to deal with the latter, and only the brighter objects found usually in the N.G.C. are considered. At the same time, areas containing many of these nebulæ are shown separately in the diagrams. Such areas contain usually one or two small spirals and a number of faint, featureless objects, round or oval in form, and often much brightened towards a central point. It is assumed in *Lick Observatory Publications*, 13, that all the small nebulæ appearing on plates exposed in regions containing the specifically spiral nebulæ are themselves spiral in form, and on this assumption Curtis bases an argument as to the total number of spirals in the sky. He



Galactic Distribution of the Spiral Nebulæ < 2′ Diameter.

arrives at a probable number of 722,000 which is certainly much too high, even if it is allowed that all the small nebulæ are actually spirals. His argument is invalidated by the fact that the distribution of this class of nebulæ is by no means uniform in the sky, and one quarter of the galactic sphere is almost devoid of these objects. Recent work at Helwân and at Mount Wilson has shown conclusively that the assumption that all the small nebulæ are actually spiral in form is not justified by observation. It was pointed out by the writer in the *Observatory* for 1920 November, that there were good reasons for the conclusion that a great number of the small nebulæ were of the nature of nebular nuclei, some of them being globular or ovoid in form, and thus representing nebulæ in their primitive form before the development of spiral arms, others being probably of the same nature as the condensations found in the spiral nebulæ. The small nebula N.G.C. 221, near the Andromeda Nebula, is a good instance of the first class. It is exceptionally bright and about 2′ in diameter, with no trace of spiral structure. Its radial velocity is the same as that of the Andromeda Nebula (– 300 km./sec.), and it obviously forms part of the system at approximately the same distance. Another example is N.G.C. 4486, which has been photographed at Mount Wilson with the 100-inch reflector. This nebula also is about 2′ in diameter, circular in appearance, with a strong increase of light up to the centre, but consisting of a perfectly smooth, structureless nebulosity. No trace of anything suggesting star discs is to be seen, although the scale of the photograph and the light grasp of the instrument would surely be great enough to reveal a mottling in the nebulosity if stars were present. The spectrum is of normal "G" type such as is usually found in spirals, and we must therefore look for some other explanation of the spectrum than that these nebulæ are stellar in their constitution. We may therefore conclude that the existence of a class of nebulæ which are definitely not spiral in form, although of the same spectral type and general distribution, is well established. It may be added that the existence of such nebular nuclei is postulated by Dr. Jeans' theory of spiral development, and the observed forms from circular to lenticular in projected outlines are in very good agreement with his theoretical forms. It must not be concluded from the foregoing that spirals of less than 2′ diameter are rare. In number they are considerable, and their spiral form down to 30″ diameter can be made out under magnification in negatives on the astrographic scale. The question as to how many spirals smaller than 30″ can be detected must be answered by the work of the large American reflectors. The small nebulæ found in the plates exposed and stored at observatories devoted to this kind of work fall naturally into four divisions:—

1. Definite spirals.
2. Much elongated nebulæ (or spindles), which are presumably spirals, as the spiral planes must have been developed.
3. Spheroidal nebulæ, either round or ovoid, in which no trace of spirality can be detected.
4. Small, faint nebulæ, only distinguishable from star discs under magnification, the edges of the discs being ill defined.

The number of definite spirals under 2′ diameter yet observed is not so numerous as classes 3 and 4, but we must obviously include the elongated spindle nebulæ with them, as such thin planes indicate a spiral form. For the purposes of this paper, the two classes are therefore taken together. A difficulty in arriving at the number and distribution of these and the nebulæ of classes 3 and 4 is encountered in the absence of definite information as to the area from declination $-45°$ to $-90°$. The only source yet available is the Franklin Adams plates of this region, and the scale of these is too small to give much reliable evidence. In the distribution diagrams this area is marked off by a dotted line: the South Galactic Hemisphere is of course principally affected, although there is a small portion of the North Galactic Hemisphere included in the area. At the same time the south polar region is not found to contain many nebulæ of any type, either in the N.G.C.s or in the F.A. plates. Some large spirals and what appear to be globular nebulæ or small spirals are to be found in the region lying round $\lambda 230°$, $\beta -40°$.

Galactic Distribution of the Small Spirals and Spindles (under 2′ diameter).

In the North Galactic Hemisphere about 136 objects are listed in *Lick Obs. Publications*, 13, the *Mount Wilson Contributions*, and the *Helwân Bulletins*, all of which are definitely described as spirals and spindles, doubtful objects being excluded. This number is smaller than the similar objects of 2′ to 5′ from the same sources. Following the system adopted in the paper on "The Galactic Distribution of the Spiral Nebulæ, with special reference to Galactic Longitude," these have been divided into three classes according to the angles made by their planes with the line of sight, derived from recorded dimensions. The numbers come out as follows:—

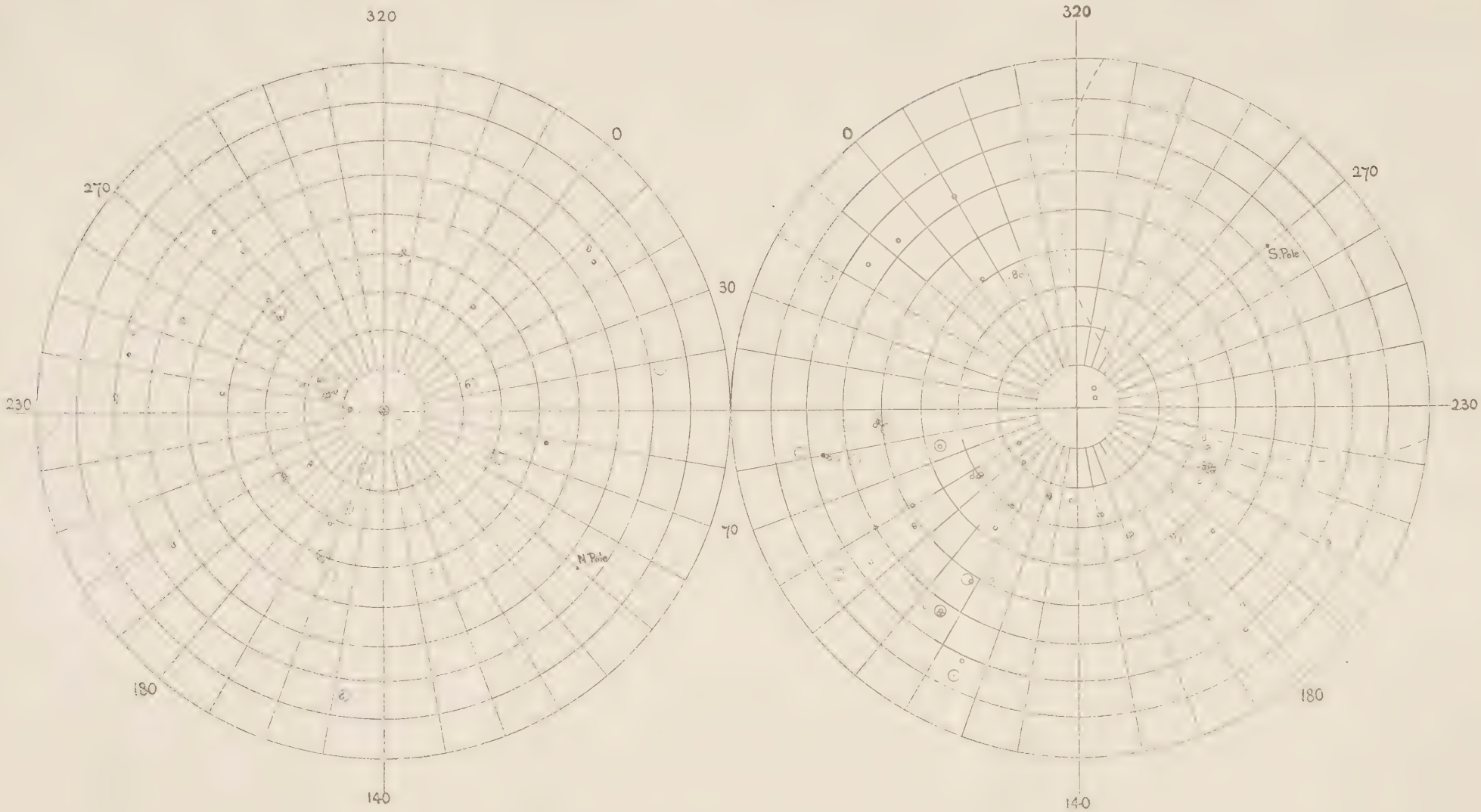
$<25°$	54
$25°-35°$	28
$>35°$	50

In the South Galactic Hemisphere 86 examples have been extracted from the lists; these are divided as follows:—

$<25°$	35
$25°-35°$	23
$>35°$	28

It will be seen that in the N.G.H. the numbers under and over the average random angle of 30° are nearly equal, while in the S.G.H. there is a somewhat greater divergence. If we compare these figures with those given in the previous paper we get the following results:—

	10′ diam. and over.	5′ and <10′.	2′ and <5′.	<2′.
1. $<25°$	18	47	119	89
2. $25°-35°$	2	15	70	51
3. $>35°$	6	21	85	78
Proportion of spirals under and over 25°–35°	3 : 1	2·2 : 1	1·4 : 1	1·12 : 1



Spheroidal Nebulæ ◯
Groups of Small Nebulæ ⊙

Galactic Distribution of the Spheroidal and Nuclear Nebulæ.

This seems to indicate quite definitely a progression from a large excess of small inclinations in the spirals 10′ diameter and over to almost equal proportions in the small spirals under 2′ diameter. There is no known systematic effect which would explain the difference found, and the agreement of the spirals under 2′ diameter with the larger examples in this progression affords good evidence that it is real and not spurious. The distribution of this class of small spirals in the N.G.H. is not markedly different from that of the spirals 2′ to 5′ diameter, except that they are more widespread. There are some erratics in areas which contain very few spirals, but the area between λ320° and λ50° (through 360°) in the northern galactic hemisphere is almost devoid of them, and they are not numerous in the region of the large spirals. The semi-hemisphere λ100° to λ280° contains about 85 per cent. of the total number in the N.G.H. Two groups in Virgo form a continuation of the well-marked band of spirals stretching from Ursa Major to and beyond the north galactic pole, the spirals decreasing in average diameter in the same direction. In the S.G.H. also the small spirals are more widely spread than the larger spirals, and are found extending well into the region bounded by λ320° to λ50° (through 360°), a feature not found with the larger spirals.

The Spheroidal, Lenticular, and Nuclear Nebulæ.

As far as the brighter objects are concerned, the number of these appearing in the N.G.H. is not so great as in the S.G.H. They vary very much in diameter, some only appearing as hazy star discs, others attaining a diameter of 2′. The following list of the more conspicuous is compiled from the publications of the Lick, Mount Wilson, and Helwân Observatories.

N.G.H.			S.G.H.		
N.G.C. 2968	4261	4649	N.G.C. 83	596	1395
2970	4278	4742	II. 1554	683	1399
3073	4374	4760	163	731	1400
3245	4382	4856	221	736	1407
3377	4383	5061	254	770	1427
3379	4406	5846	274	838	1510
3384	4472	5982	404	941	1638
3607	4486	6482	410	1196	2217
3923	4546	6500	474	1391	7454
			524	1379	7465
			584	1387	7778

Some of these objects are oval in projected outline, and are probably therefore in rotation in accordance with Jeans' theory of spiral development. In one or two instances a band of absorption appears along the major axis of the ellipse, indicating a flow of matter along the periphery. Especially remarkable is the grouping together of this class of objects in the same region of the sky. The most notable instance of this is the region round $\alpha = 3^h\ 32^m$, $\delta = -35^\circ\ 50'$, $\lambda = 204^\circ$, $\beta = -52^\circ$. In this

region 13 are congregated in about a square degree, all described as bright, without any definite structure, some round and some elongated. It is not unusual for a much greater number of small, faint nebulæ to be found on a single plate, but the presence of so many bright objects together is quite exceptional. It is also to be remarked that in the majority of instances one or two nebulæ appear in these groups which are definitely spiral, almost always of less than 2′ diameter. This points to the nebular areas being much farther removed from our position in the galaxy than the large spirals.

The information we have up to the present with regard to this class of nebulæ is necessarily incomplete, and the present paper is only intended to indicate general results, but there seems to be good evidence that there is a preponderance of these objects in the S.G.H., especially as there is a considerable area of this hemisphere still unexplored. On the other hand, the number of actual spirals of all dimensions in the N.G.H. is in excess of those in the S.G.H. in the proportion of 3 to 2. If these spheroidal nebulæ are the primitive nuclei from which the spirals are developed, it appears that the N.G.H. is in a more advanced stage of development than the S.G.H. as far as the spirals are concerned. This probably may have an important bearing on the age of these formations generally in the two hemispheres.

The H and K Lines of Calcium in O-type Stars.

By J. S. Plaskett, F.R.S.

Introduction.

The curious behaviour of the H and K lines of calcium in the early B-type stars has been commented on by many observers. They were first observed as peculiar by Hartmann * in the spectroscopic binary star δ Orionis, in which the calcium lines are sharp and narrow and give a constant velocity, nearly the same as the velocity of the binary system, while the other star lines of hydrogen, helium, etc., are diffuse and broad, and oscillate over a large range. Many other stars showing a similar effect have been discovered at the Yerkes, Lowell, Allegheny, and other observatories, and the whole available material was summarised and discussed by Reynold K. Young † in the publications of this observatory. He reached the general conclusions, from the observations then at hand, that the effect was confined to the high temperature spectroscopic binary stars, probably not appearing "later" than type B3, and that the velocities from the sharp calcium lines, though not always constant, varied through a smaller range than the velocities from the other star lines. In contradiction to the hypothesis of Hartmann and Henroteau ‡ that the calcium absorption was due to an interstellar cloud of calcium vapour between star and earth, Young suggested as

* *Ap. J.*, 19, 268.
† *Pub. D.A.O.*, 1, 219.
‡ *Jour. R.A.S.*, 15, 62.

a working hypothesis that the cloud surrounded the star although probably outside the reversing layer which produces the diffuse star lines.

The phenomenon was extended by Miss Heger* of the Lick Observatory to the D lines of sodium, which were found to be present as narrow, sharp, stationary lines agreeing in velocity, within the errors of observation, with that given by the calcium lines in the spectroscopic binaries δ Orionis and β Scorpii, two outstanding examples of stationary calcium lines. Similar results were later obtained by Miss Heger† for some other B-type stars with sharp H and K lines. There seems to be no doubt from this work and from similar measures on some of the O-type stars, to be detailed later, that the absorbing cloud which produces the calcium lines must be similarly the cause of the sharp D lines.

New Observational Material.

Neither of the two hypotheses advanced, interstellar or surrounding clouds, appears to fit all the observed facts, and objections can be adduced to each. Consequently further observational material is desirable, and this is offered by the spectroscopic investigation of the O-type stars now nearly completed at Victoria by the writer. The absorption line O-type stars, as has been shown by H. H. Plaskett,‡ form a continuous sequence beyond the B's on the side of higher temperature through about five spectral subdivisions. As Young§ has shown that the characteristic stationary H and K lines do not appear lower in the spectral sequence than B3, it was of interest to see if there was a limit at the other end of the scale, particularly as R. H. Fowler and Milne‖ have shown that stellar H and K should disappear about type Bo or when the lines of enhanced helium appear. As will be seen shortly, these characteristic H and K lines, and also the sodium D lines, so far as investigated, appear not only in all the subdivisions of the absorption O-types, but are present in every example of each subdivision observed. They also show strongly in all the emission O-types (Wolf-Rayet stars), for which a long enough exposure was given to impress sufficient continuous or emission spectrum in that region for differentiation of the H and K. It was previously supposed that the stationary H and K were confined to spectroscopic binaries, but they are always present in the O-type stars whether of variable or constant velocity. Further, there is the curious fact that the velocity obtained from H and K, and from the D lines, is generally constant and almost invariably differs considerably and sometimes greatly from that given by the other lines.

In order to exhibit these phenomena clearly, the radial velocities obtained both from the H and K of calcium and from the other lines are grouped in the following table in different classes, first according to the positions of the stars in the sky and later with respect to the differences in these velocities.

* *L.O.B.*, 10, 59.
† *Idem*, 10, 141.
‡ *Pub. D.A.O.*, 1, 363.
§ *Idem*, 1, 219.
‖ *M.N.*, 83, 403.

Instruments and Methods.

The one-prism form of the Victoria spectrograph * was used in obtaining these velocities, two cameras of medium and short focus being employed giving respectively linear dispersions of 29 A. and 49 A. to the millimetre at Hγ. The higher dispersion was used on the brighter stars, but in the majority of cases, and exclusively with stars fainter than the seventh magnitude, the lower dispersion was employed in order to avoid excessive exposures in poor seeing. While the accuracy of radial velocities obtained is, with sharp lines, approximately proportional to the dispersion, the gain with the higher dispersion with the broad diffuse lines usual in this spectral class is not sufficiently great to compensate for the doubled observing time. With such lines and small dispersion one should not expect exact agreement among the different plates even when the star's velocity is constant, and variations as high as 10 km. per sec. or even more in individual plates need not be considered as indicating real differences in velocity.

The calcium velocities were obtained from the measures of both H and K in almost all cases, for although Hε is near H it is generally not so broad and strong that the much narrower and stronger H is not clearly defined against it even with the smallest dispersion used. Occasionally the velocity measures from the two lines were weighted equally, but generally, owing to better quality, the K line was given greater weight. The stellar radial velocities were obtained from the measures of the best defined of the usual fuzzy lines of the O-type stars. The lines present are due to the elements, hydrogen (the Balmer Series), enhanced helium (4686, the Pickering Series and the companions † to the Balmer lines), ordinary helium and enhanced nitrogen, silicon, carbon, oxygen, with occasional magnesium 4481. The number and quality vary greatly in different stars, the number ranging from 2 or 3 to about 20, while the quality is generally poor, the lines being broad and diffuse. Only in two stars are the lines sharply defined. Consequently the probable error of measurement is high, on the average about 5 km. per second for a single plate. The difficulty of obtaining reliable velocities is increased by the fact that considerably more than half of the stars of absorption O-types are variable in velocity. In these cases the stellar velocity, where an orbit has not been obtained, is taken as the mean of the measured velocities, and is represented in the tables by being given only to the nearest whole number in kilometres per second.

In the tables, the first column contains the Durchmusterung number of the star, the second and third the right ascension and declination for the epoch 1900, the fourth the visual magnitude, and the fifth and sixth the spectral types according to the Harvard classification and H. H. Plaskett's ‡ proposed scheme. Data about the radial velocities are given in the seventh, eighth, and ninth columns for the star lines, and in the tenth, eleventh, and twelfth for the calcium lines. In the seventh and tenth columns are given the number of spectra from which the mean velocities in the eighth and eleventh columns are derived,

* *Pub. D.A.O.*, **1**, 81 ; also *Ap. J.*, **49**, 209, 1919.
† *Pub. D.A.O.*, **1**, 335. ‡ *Idem*, **1**, 363.

while the probable errors are given in the ninth and twelfth. As previously indicated, where the star has a variable velocity this is shown by giving the velocity to the nearest whole number unaccompanied by a probable error. As generally the velocity range is not great, this assumed value is probably close enough for statistical purposes, and can only be improved upon by a much greater number of observations, which do not seem at present to be justified. The star and calcium velocities assumed constant are given to tenths of a kilometre, accompanied by the probable error of the *mean velocity* whenever a sufficient number of plates are available to give it any meaning. The size of this probable error gives some information as to the reliability of the velocities and indirectly as to the quality of the lines for measurement. The thirteenth column contains the component of the solar velocity in the direction of the star, computed from the position of the solar apex at R.A. 270°, Dec. +30° and velocity 20 km. per second, while the fourteenth column is reserved for remarks.

Discussion of Velocities.

As will be noted from the table, the stars concerned are subdivided into various groups according to position in the sky, with two groups at the end containing comparisons of velocities. It will be desirable to discuss briefly the velocities in each of these groups in turn.

First Group—Perseus Region.—It will be noted that four of the six stars in this group are variable in velocity and in two of them the calcium velocity also appears to vary. The range, however, in all these cases is not great, and the mean velocities probably do not greatly differ from the true ones. There appears to be a group motion of approach of these stars of nearly 50 km. per second, about 30 km. greater than that of the calcium vapour. The calcium lines in the stars in this region are not so strong and sharply defined and the velocities differ more markedly from the solar component than elsewhere.

Second Group—Monoceros Region.—Except for λ Orionis this group is a compact one surrounding the very massive star 6°1309 whose orbit and dimensions were recently presented here.* Six new O-type stars previously classified as B's were found in this group, and there seems to be unmistakable evidence of a group motion of recession of the stars of 35 to 40 km. per second. The calcium vapour is moving in the same direction but with 20 km. lower speed. While there are small differences in the calcium velocities in different stars, partly due to errors of observation, the general trend as will be noted is toward close agreement with the reflex of the solar motion.

Third Group—Cygnus Region.—The first subdivision of this group, containing six stars—four O-type, two B-type, four of which are binaries —forms, within an area only about half a degree square, an interesting, compact group of high temperature stars all of which have sharp stationary calcium lines. The velocities from these lines, even allowing for errors of observation, seem not to be the same for different stars, and further seem to average about 4 km. per second less negative than the solar component.

* *M.N.*, 82, 447.

Perseus Region.

Star. B.D.	R.A.	Dec.	Mag.	Type. Har.	Type. Rev'd.	Radial Velocities. No.	Star.	P.E.	No.	Calcium.	P.E.	Solar Comp.	Remarks.
	h m							±			±		
62° 2363	0 0·8	63° 07′	7·36	B	O6e	11	−64·1	1·3	11	−20·4	1·2	− 8·9	Emission H.
61 92	0 23·2	61 57	8·6	B	B1	4	−51		3	− 8·0		− 8·0	
62 259	1 24·6	62 51	7·46	Bop	B2	4	−40·1		4	−20·9		− 6·0	
40 501	2 16·7	41 02	7·7	B	O9	11	−37		11	−14·3	0·6	+ 0·9	
58 467	2 19·5	58 25	8·04	Oe	O5e	10	−54		8	−26		− 3·3	
56 693	2 35·5	56 28	8·4	Oe	O6e	6	−41		3	−22		− 2·4	

Monoceros Region.

9 879	5 29·6	9 52	3·66	Oe5	O8	8	+36·1	0·6	8	+12·5	1·5	+15·1	λ_1 Orionis.
9 879	5 29·6	9 52	5·56	Oe5	B1	8	+36			+16·1	1·1	+15·1	λ_2 Orionis.
11 1204	6 25·6	11 19	5·83	Bo	B2e	7	−20		6	+22·7	0·9	+14·9	
5 1282	6 26·6	5 06	7·66	B2	O8	5	+42		2	+17·4		+16·2	
4 1302	6 27·0	4 53	7·14	B2	O5	4	+41·4		4	+16·0		+16·2	
6 1303	6 31·1	6 10	7·3	B2	O8	6	+42·7	1·3	5	+15·2	1·3	+16·0	
6 1309	6 32 0	6 13	6·13	Bop	O8e	30	+23·9	1·4	28	+16·0	1·3	+16·0	Very massive star.
5 1334	6 32·5	5 03	6·16	B1	B1	6	+36		3	+17·2		+16·2	
10 1220	6 35·5	9 59	4·68	Oe5	O7	9	+34·3	0·9	7	+16·2	0·8	+15·1	S Monocerotis.
6 1351	6 36·6	6 27	6·20	B2	O6	8	+31		5	+15·5	1·8	+15·8	
4 1414	6 38·3	4 02	5·78	Bo	B1	5	+34·5	1·0	4	+20		+16·3	

Cygnus Region—Absorption O-Type.

35 3930N	19 59·8	35 45	7·2	Oe	O5e	12	−17		11	−11·1	1·2	−18·0	
35 3930S	19 59·8	35 45	7·5		O9	6	+ 9		5	−13·5		−18·0	
35 3949	20 1·9	35 19	8·2	B	O6	7	+ 3		5	−14·4		−18·0	
35 3952	20 2·2	35 24	7·30	B	B1	5	−22		4	−13·0		−18·0	
35 3953	20 2·2	35 31	7·01	Op	O9e	8	−13·4	2·1	7	−16·7	1·4	−18·0	
35 3955	20 2·2	35 32	7·8		B1	7	− 8·8	1·1		−14·0	1·9	−18·0	

Emission Wolf-Rayet 1*-Type.*

35 4001	20 6·5	35 53	7·8	Ob					5	−15·9	0·9	−17·6	Typical Ob.
35 4013	20 8·1	35 54	7·94	Oa					3	−16·5		−17·6	Typical Oa.
37 3821	20 8·4	38 03	7·44	Ob					7	−18·3	0·8	−17·6	
36 3956	20 10·8	36 21	7·94	Oa					3	−17·1		−17·5	
36 3987	20 13·3	37 07	7·97	Oc					2	−20·3		−17·5	Typical Oc.
43 3571	20 17·1	43 32	6·83	Ob	O6e	9	+ 5		7	−18·6	1·3	−17·1	

Cygnus, Cepheus, Lacerta.

44 3639	20 53·1	44 33	6·01	Oe5	O6	33	− 5·8	0·7	33	−11·3	0·3	−16·0	Orbit.
43 3877	21 14·8	43 31		Oe5	O8	27	+ 4		26	−12·8	0·6	−15·1	A Cygni.
56 2617	21 35·9	57 02	5·64	Oe5	O6	31	− 1·0	2·0	25	−18·5	0·6	−13·9	Orbit.
61 2246	22 2·1	61 48	5·17	Oe5	O9	16	−10·9	0·9	16	−12·8	0·6	−12·7	
58 2402	22 8·1	58 56	5·19	Od	O6e	15	−73·6	2·2	15	−14·4	0·6	−12·6	λ Cephei.
38 4826	22 34·8	38 32	4·91	Oe5	O9	7	− 7·9	0·3	7	−19·0	0·5	−11·2	10 Lacertæ.

Miscellaneous.

Star. B.D.	R.A.	Dec.	Mag.	Type. Har.	Type. Rev'd.	Radial Velocities. No.	Star.	P.E. ±	No.	Star.	P.E. ±	Solar Comp.	Remarks
	h m	° ′											
52° 726	3 48·1	52 21	6·70	Oe5	O9	8	− 8·6	1·6	7	− 3		+ 0·9	
34 980	5 9·7	34 12	5·81	Bop	O9	9	+59·0	0·8	8	+13·3	1·1	+ 8·2	
−10 1892	7 4·6	−10 11	6·20	Oe5	O7	5	+64·9	2·6	3	+31		+18·1	
17 1623	7 31·4	17 07	7·7	Oe5	O8e	5	+25·4		5	−13·1		+12·3	
18 4276	19 47·9	18 25	6·29	Oe	O7e	14	+17·8	1·3	12	− 5·0	0·9	− 9·1	9 Sagittæ.

Stars with Large Differences.

62 2363	0 0·8	63 07	7·36	B	O6e	11	−64·1	1·3	11	−20·4	1·2	− 8·9	Bright H.
40 501	2 16·7	41 02	7·7	B	O9	11	−37		11	−14·3	0·6	+ 0·9	
34 980	5 9·7	34 12	5·81	Bop	O9	9	+59·0	0·8	8	+13·3	1·1	+ 7·5	Sharp lines.
9 879	5 29·6	9 52	3·66	Oe5	O8	8	+36·1	0·6	8	+12·5	1·5	+15·1	λ_1 Orionis.
6 1303	6 31·1	6 10	7·3	B2	O8	6	+42·7	1·3	5	+15·2	1·3	+16·0	
56 2617	21 35·9	57 02	5·64	Oe5	O6	31	− 1·0	2·0	25	−18·5	0·6	−13·9	
58 2402	22 8·1	58 56	5·19	Od	O6e	15	−73·6	2·2	15	−14·4	0·6	−12·6	λ Cephei.

Comparison—Stellar, Calcium, and Sodium.

Star.	R.A.	Dec.	Mag.	Type. Har.	Type. Rev'd.	Radial Velocities. No.	Star.	P.E. ±	No.	Calcium.	P.E. ±	No.	Sodium.	Solar Comp.
	h m	°												
ξ Persei	3 52·5	35 30	4·05	Oe5	O7	44	+65		44	+15·4	0·6	2	+13·4	+ 6·2
9 Camelop.	4 44·1	66 10	4·38	Oe5	O8	79	+ 6		79	− 2·2		1	− 0·6	− 3·0
λ_1 Orionis	5 29·6	9 52	3·66	Oe5	O8	8	+36·1	0·6	8	+12·5	1·5	3	+16·9	+15·1
ι Orionis	5 30·5	−5 59	2·87	Oe5	O8	107	+21·3	0·9	29	+ 29		1	+23·8	+18·1
S Monoc.	6 35·5	9 59	4·68	Oe5	O7	27	+34·3	0·9	26	+16·2	0·8	1	+11·1	+15·1
A Cygni	21 14·8	43 31		Oe5	O8	27	+ 4		26	−12·8	0·6	2	−17·1	−15·1
λ Cephei	22 8·1	58 56	5·19	Od	O6e	15	−73·6	2·2	15	−14·4	0·6	2	−16·5	−12·6

The second subdivision of the Cygnus group is perhaps the most interesting of all, consisting of emission O-types, Wolf-Rayet stars. As is well known their spectra consist of broad emission bands, overlapping one another and superposed on a weak continuous spectrum. There are no traces of either broad or narrow stellar absorption lines in the first five of these, and consequently not even an approximate value of the stellar velocity can be given, as the breadth and mixing of the emission bands with uncertainties of identification make their positions indeterminable within two or three Ångströms. Absorption appears entirely foreign to the atmospheres of these stars, although the narrow, sharp H and K are superposed on the spectrum of every Wolf-Rayet star photographed as soon as sufficient background of either emission or continuous spectrum is present to enable them to be differentiated from it. The last star, though classified as Ob and showing very broad and strong emission at 4650, also has hydrogen and enhanced helium absorption which shows variable velocity. The calcium velocities from this group, which is two or three degrees north-east of the preceding, also differ somewhat among themselves but are in closer agreement with the solar component.

The third subdivision of the Cygnus group, which in this case extends into Cepheus and Lacerta, contains six bright stars, three of them binaries, and there are here as elsewhere considerable differences, especially marked in λ Cephei, between the stellar and calcium velocities. The latter, as the probable errors show, are well determined in these bright stars, where the spectra were mostly of the higher dispersion, and there seems to be no doubt of the reality of the small irregular differences between the calcium velocity and the solar component although the average is nearly the same.

Fourth Group—Miscellaneous.—This comprises a number of O-type stars which cannot well be included in any of the previous groupings. These also show marked differences between stellar and calcium velocities and probably real differences between the latter and the solar components.

Fifth Group—Stars with Large Differences.—Stars from the preceding groups which show the most marked differences between stellar and calcium velocities are here brought together. A simple comparison of the velocities in the two columns with their probable errors shows the impossibility of believing that the calcium vapour is a part of the permanent stellar atmosphere and is moving with the star.

Sixth Group—Comparison of Stellar, Calcium, and Sodium Velocities.—The velocities given by the D lines of sodium were obtained only for a few of the brighter stars, on account of the long exposures required with the three-prism dispersion on the slower panchromatic plates. However, sufficient have been secured to show the close agreement, when observational errors are allowed for, between sodium and calcium velocities. The first, second, fourth, and sixth stars of the list are binaries, and in the first three of these the calcium velocities also vary through a small range, so that small differences between calcium and sodium may be expected. The agreement of these two elements when compared with the widely different velocities from the stellar lines seems

to point unmistakably to a common origin or cause of the calcium and sodium absorption. It should be noted, however, that the strength of the D lines varies over a wide range in the stars observed. In the first two and last two stars of the group they are very strong and sharply defined, while in the other three they are quite weak and difficult to measure, a much greater variation of character than occurs in the calcium lines, although some peculiarities in the latter were pointed out in the Perseus group, and they are much weaker than normal in θ Orionis and ι Orionis. These differences should be kept in mind in any proposed hypotheses.

Early Hypotheses of Origin.

As indicated in the introduction, the two most promising hypotheses of the origin of these sharp calcium and sodium lines were: (*a*) the presence of an interposing interstellar cloud of calcium vapour between earth and star, and (*b*) a surrounding cloud belonging to the star though probably outside the normal stellar atmosphere.

The principal objection to the first hypothesis has developed with advancing knowledge of astronomical physics. Such an interstellar cloud, assumed to be anywhere between earth and star, must be quite widespread, and, there being no reason to suppose its radial extension smaller than its lateral, it must be exceedingly tenuous, for the most recent evidence seems to demonstrate the almost complete absence of any general absorption. At the very low temperature of interstellar space it is difficult to see how the calcium and sodium in such a tenuous cloud can be in the form of a gas, still less how the calcium can be ionized, a condition necessary to produce the absorption H and K lines. Unless ionized calcium atoms can exist in free space and unless the neutral sodium atom remains in the gaseous state, such interstellar clouds will not give absorption lines, and the first hypothesis, in its original form, fails.

The evidence furnished by the radial velocities of the O-type stars from the stellar and calcium lines, as tabulated above, seems to be even more certainly against the second hypothesis of a surrounding cloud belonging to and presumably moving with the star. The differences between stellar and calcium velocities in the observations available when this hypothesis was formulated were, in general, sufficiently small to be explained by wave-length effects due either to errors in the laboratory values or more likely to displacement or asymmetry of the diffuse star lines which may be ascribed to pressure or other effects. But the consistently large velocity differences in so many of the O-type stars decisively puts this explanation out of court. A pair of examples from the fifth group will suffice to demonstrate this. The star 34° 980 has narrow, sharply-defined stellar lines, twenty or more being measured on each plate with excellent internal agreement giving a constant velocity of *recession* of 59 km. per second, 46 km. greater than that of the calcium lines. λ Cephei, on the other hand, has an approximately constant velocity of *approach* of 74 km. per second, 59 km. greater than that of the calcium vapour. It is beyond the bounds of credibility to assume, with this and similar new evidence, that any cause but difference of

velocity can displace the beautifully sharp star lines of 34° 980 two-thirds of an Ångström to the *red* of the calcium lines while a presumably similar cause can shift the broader lines of λ Cephei nearly a whole Ångström to the *violet*.

If it is noted in addition that in the Perseus region the stellar velocities are 30 km. more negative, and in the Monoceros region 20 km. more positive, than the calcium, to say nothing of the differences in the rest of the table, and particularly that the calcium velocities in any one region are nearly the same and agree approximately with the solar component reversed, while the stellar velocities are very varied, it can no longer be assumed that the calcium and sodium vapour is connected with, and moving with these stars, and the hypothesis of an individual surrounding cloud belonging to each star must also be abandoned.

Analysis of Conditions.

As a preliminary to formulating a new hypothesis, let us review carefully the evidence now available and see in what direction it tends. Consider first the character of this absorption, particularly the calcium H and K, as the D lines have only been examined in a few stars. The H and K lines persist, as uniformly strong, sharp, and narrow lines, constant or nearly constant in position, through four subdivisions of the B-type—B3 to B0—and in practically every example of all five subdivisions of the O-type—O9 to O5. What is still more striking, the H and K lines, with similar characteristics, are superposed on the emission bands and on the continuous spectrum of the three Harvard subdivisions, Oa, Ob, Oc, of the Wolf-Rayet stars, none of which show a trace of any other absorption. A spectrum of the planetary nebula N.G.C. 2392, obtained by H. H. Plaskett, clearly shows sharp H and K appearing in the spectrum of the nucleus, and it is hence probably present in all planetary nuclei. Finally, as is well known, sharp H and K, and also sharp D_1 and D_2 wherever observed, appear in some stages of the development in the spectra of the Novæ. It seems hardly possible to suppose that these lines, and these only, should remain constant in character over so wide a range of celestial objects, in which the other lines are appearing, changing in character and position, and disappearing in the most bewildering way, unless they are due to some common origin and are produced in some uniform way independent of the widely different conditions prevalent over such a wide range of spectral types.

A similar conclusion is reached by a consideration of the velocities already tabulated. Except perhaps in the Perseus group, the calcium velocities in any one group differ only slightly from one another and from the solar component, while the stellar velocities cover a wide range and show no relation to the solar motion. Expressed otherwise, the space velocities of the calcium vapour, relative to the system of stars from which the solar motion is derived, are generally very small, and there seems to be a marked community of motion in any one group. This consideration as well as the preceding, of the constancy of character of the calcium lines over so wide a range of objects, seems inevitably to

point to a common origin of the absorbing material, and we appear to be led back to the idea of diffuse and widely-extended clouds with small space velocities. The small deviations of the cloud velocity at different stars may be due to small local motions in such clouds, and are no greater than are known to exist in different parts of the Orion Nebula.

Proposed Hypothesis.

The considerations just adduced can hardly be satisfied by any other origin of the calcium and sodium absorption than a fairly widespread cloud of tenuous material through which the starlight shines without appreciable general absorption, but with strong selective absorption at H and K, and at D_1 and D_2. Such material can hardly be local and peculiar to each star, else it should more nearly partake of the stellar motions, and not show, as happens, an entire independence of them. Further, in such case, the H and K lines in neighbouring stars and in stars of so wide a range in type would not be likely to show community of motion and identity of character, as is unmistakably and generally exhibited.

It seems necessary to assume from the evidence that the matter giving the sharp H and K and the sharp D absorption must be of fairly general distribution, and extend over wide areas in the sky. Although the evidence is chiefly confined to regions in and near the galaxy, there is no reason to suppose it is a purely galactic phenomenon. Indeed, one undoubted example, ρ Leonis, observed by Mr. Harper, showing a difference of over 30 km. between calcium and stellar velocities, has a galactic latitude of 53°. This wide distribution, and hence probably great depth with the absence of any general absorption, indicates extreme tenuity of the absorbing matter. In addition, the velocity evidence shows that it must be nearly stationary with respect to the local stellar system, a natural conclusion if its wide distribution is assumed.

The difficulty of excitation of this widely distributed, tenuous cloud of matter still exists, if the cloud is assumed to lie in free space between us and the stars, and in this form it is practically identical with Hartmann's original hypothesis. I am indebted to my son, H. H. Plaskett, in a discussion of this objection, for the suggestion that removes it. All that is necessary is to assume that the absorbing material is near to or envelopes the stars, which is probable from its wide distribution, and in this form it combines the two original hypotheses of interstellar and surrounding clouds. Radiation from the very high temperature stars, to which the phenomenon is confined, should readily ionize sufficient calcium in the neighbourhood to give the H and K lines—sharp and narrow by reason of the low density of the cloud—while the same cause would equally well explain the sodium lines. This ionized calcium and active sodium is unlikely to be set in rapid motion owing to the approximate balance between gravity and radiation pressure on atomic material. This is in agreement with observation, as the slight differences in the calcium velocities for different stars may be due either to local motions in the cloud, as in the Orion

Nebula, or to slightly varying motions of the active material for stars of different mass, temperature, and velocity.

The probability that stellar radiation is competent to produce the excitation necessary to give the observed absorption is much increased by its analogy with the results of Hubble's* recent important work on the diffuse galactic nebulæ. He conclusively shows that the luminous nebulæ almost invariably have O or early B-type stars enmeshed in or close to the nebulosity, stars from B1 to B3 giving luminosity of a continuous character, while Bo and the O-types produce emissive luminosity. We can hardly escape the conclusion, especially as this luminosity in general decreases with distance from the stars according to the inverse square law, that it is due to excitation from the stars, and thence, by analogy, that the calcium and sodium absorption, probably requiring considerably less energy, is similarly produced, especially as almost exactly the same spectral types are concerned in the two cases. It is a curious and perhaps significant fact that several of the O-type stars which show sharp H and K are involved in luminous nebulæ. 34° 980, an O9 star with sharp lines and 46 km. difference between stellar and calcium velocities, is involved in I.C. 405. Four early O's in Monoceros with 20 km. difference between stellar and calcium velocities are involved in N.G.C. 2237. The Wolf-Rayet star 37° 3821 also giving sharp H and K of nearly zero space velocity, but with no other absorption, is involved in N.G.C. 6888, and there are other instances. However, sharp H and K frequently appear where there is no visible luminosity, and the calcium clouds are probably more widespread than the luminous ones.

It seems certain that the material must be extremely attenuated, else, as previously stated, the great depths would produce noticeable absorption of starlight. It is possible, then, that the difference between the luminous nebulosity and the calcium clouds is mostly a matter of density. It is probable that much less material or fewer active atoms are required to produce absorption than emission, and this would serve also to explain why the H and K lines have not been seen bright. In this connection it is perhaps well to direct attention to a suggestion of Saha's† that the calcium producing the sharp H and K is emitted by the stars themselves, although he was not aware of the presence of sodium, which behaves similarly but is not so readily explained on his hypothesis. If the material forming these clouds was emitted from the stars generally, its widespread distribution and small space-velocities would be naturally explained.

A very pertinent question results from the possibility of the complex composition of the cloud. Why do only calcium and sodium appear as absorption, and why are other elements not present? It is significant that the lines appearing are the first lines of the principal series of neutral sodium, an alkali metal, and of ionized calcium, an alkaline earth metal, and that the latter has a similar outer electron configuration and presumably similar chemical properties to the alkali metal potassium, pointing to the possibility that conditions in the cloud favour only the

* *Mt. Wilson Contributions*, Nos. 241, 250.
† *Nature*, 107, 488, 1921.

alkali metals and atomic analogues. In such case, by the displacement law,* we should look for additional lines among the first lines of the principal series of—(*a*) the neutral atoms of the alkali metals; (*b*) the singly-ionized atoms of the following group of the alkaline earths; (*c*) the doubly-ionized atoms of the next group, and so on. Group (*c*) and later groups may be ruled out as their principal lines are in the far ultra-violet, and we need only consider (*a*) and (*b*)—the alkali and the once ionized alkaline earth metals. The following table gives the principal series lines in each group:—

Alkali Metals.		Alkaline Earth Metals.	
Metal.	Principal Series Lines.	Metal.	Principal Series Lines.
Li	6707·8, 3232·6	Mg+	2795·5, 2802·7
Na	5889·9, 5895·9 (D_2, D_1)	Ca+	3933·7, 3968·5 (K, H)
K	7664·9, 7699·0	Sr+	4077·7, 4215·5
Rb	7800·3, 7947·6	Ba+	4554·0, 4934·1
Cs	8521·1, 8943·5	Ra+	3814·4, 4682·2

Bearing in mind that stellar spectra have only been investigated carefully between the limits of about 3900 to 6650, the only principal lines that could be seen in the above elements would be Ca—H and K; Na—D_1 and D_2; Sr—4077·7, 4215·5; Ba—4554·0, 4934·1; and Ra—4682·2. The Li line 6707·8 is outside the region well rendered on ordinary panchromatic plates. The percentages of relative atomic abundance of these elements in the earth's crust† are Ca 1·503; Na 2·028; Ba 0·0098; Sr 0·0065; Ra infinitesimal. Barium and strontium are less than one two-hundredth as abundant as calcium and sodium on the earth. If, as is reasonable, the relative distribution throughout space is similar, it would be remarkable if the lines of strontium and barium, present in infinitesimal proportions in a very tenuous cloud, should appear. Even if they were present they would probably be too faint and narrow to show in the small dispersion possible in stellar spectra. The fact noted above that the D lines were scarcely visible in three of the stars observed may be cited in support of the non-appearance of other cloud lines. This difference in the strength of the D lines and possibly also of H and K in different stars may be ascribed to different relative concentrations of the material or different exciting powers of the involved stars.

The above hypothesis seems satisfactorily to explain all the observational material from the O-type stars, and so far as can be seen none of the earlier data introduce any insuperable difficulties with this hypothesis. A partial oscillation of the calcium lines in the binary period, which is present in some of the B-type binaries, or the appearance of double calcium lines, suspected in two or three instances, may readily be explained by the combination of the stationary lines with calcium lines in the stellar atmosphere, and this supposition is substantiated by the recent theoretical work of R. H. Fowler and Milne,‡ in which it is shown that H and K should not disappear until about type Bo or earlier. Even the case of 12 Lacertæ cited by Young§ does

* Kossell and Sommerfeld, *Ver. d. D. Phys. Ges.*, **21**, 240, 1919.
† *Pub. D.A.O.*, **1**, 375. ‡ *M.N.*, **83**, 403. § *Pub. D.A.O.*, **1**, 225.

not necessarily refuse to yield to an analogous blend effect of stationary and stellar calcium.

However, there is no intention to claim that this hypothesis is in the ultimate form or that it will explain satisfactorily all the available data. It is only tentatively advanced as a possible explanation of the curious behaviour of H and K of calcium and the D lines of sodium, not only in the early B-type binaries in which it was first observed, but more particularly in spectra of the O-type, both absorption and emission, and of both variable and constant velocity, in which these characteristic lines seem to be universally present. The history of previous attempts at the explanation of this and other phenomena should make one hesitate about claiming it as anything but a working hypothesis.

I am greatly indebted to my son, H. H. Plaskett, for many helpful ideas and valuable assistance on the physical side of this question, and am glad to acknowledge the stimulating assistance provided by our discussions of the phenomenon and its origin. The earlier complete discussion of Dr. Young on this subject has been freely used, and I am also indebted to him and to Mr. Harper for criticisms and suggestions.

Dominion Astrophysical Observatory,
Victoria, B.C.,
1923 *November* 15.

On Gravitational Displacement of Solar Lines.

By C. E. St. John, Assoc. R.A.S.

The quantity of data bearing upon the question of gravitational displacement of the Fraunhofer lines now at our disposal at the Mount Wilson Observatory seems to justify a new discussion. The differences between the wave-lengths in solar and terrestrial spectra depend upon complicated conditions, and it has been the aim to accumulate the data in sufficient quantity and variety to make it possible to disentangle the causes of the red displacements. The following causes are considered—pressure, anomalous refraction, motion in the line of sight, gravitational shift, and differential scattering.

The differences between displacements, sun − vacuum arc for the lines of two pressure groups, and of the same mean wave-length, divided by the difference in pressure coefficients per atmosphere gives the pressure in the sun's reversing layer. For some 200–300 lines for which the data are now available the results are as follows:—

TABLE I.

Groups.	Region.	No. of Lines.	Pressure in Reversing Layer.
a–*b*	3800	41	−1·0 atmosphere
a–*b*	5050	92	+0·9 ,,
b–*d*	4100	62	+0·5 ,,
b–*d*	4550	37	−0·2 ,,

The difference in pressure coefficients for groups *a* and *b* is small compared to the difference for groups *b* and *d*, so that irrespective of the number of lines the straight mean +0·05 atmosphere may be taken. For *c*4, *c*5, and Ti lines it is +0·02 atmosphere. We are justified, it would seem, in assuming the pressure in the reversing layer to be a small fraction of a terrestrial atmosphere.

This eliminates from the problem considerations of pressure, and makes the lines of all elements available for the investigation. The crowded lines of the cyanogen bands, which were originally chosen because of their freedom from displacement by pressure, have been shown by King to change their relative intensities with increase in temperature, a characteristic which, while useful in estimating the temperature of the reversing layer as done by Birge, makes them unsuitable for an exacting comparison of arc and solar wave-lengths, especially of doublet and triplet combinations.

Anomalous refraction requires high density-gradients, but in an atmosphere extending to thousands of kilometres in height, with the pressure at the photospheric level a small fraction of an atmosphere, conditions favourable for the necessary high density-gradients are wanting.

The present observational material consists of between three and four hundred iron lines for which the wave-lengths obtained from the arc *in vacuo*, in the arc at atmospheric pressure, and in the solar spectrum are known with precision, and comparisons can be made with homogeneous material, as the individual lines are assigned to their respective groups in the pressure classification of Gale and Adams. A comparison of the solar wave-lengths with the laboratory wave-lengths *in vacuo* shows that the solar wave-lengths are always longer than the laboratory wave-lengths. When the calculated Einstein effect is deducted from the differences, sun - vacuum arc, the discrepancies between observed and calculated values are systematic, and in many cases exceed the errors of observation.

The results are as follows:—

TABLE II.

Comparison of Displacement (Sun – Vacuum Arc) Observed, and Calculated from Relativity Theory.

Group.	No. of Lines.	Solar Intensity.	Mean Wave-length.	Calculated.	Observed.	Obs.-Cal.		
a	17	12	3826	0·008	0·012	+0·004	0·3 km.	Down
b	24	14	3821	0·008	0·0112	+0·0032	0·25 ,,	,,
b	10	10·4	4308	0·0091	0·0113	+0·0022	0·16 ,,	,,
a	10	6	5419	0·0115	0·0112	−0·0003		
b	95	4·6	4166	0·0088	0·0072	−0·0016	0·1 ,,	Up
b	36	5·2	6294	0·0133	0·0115	−0·0018	0·1 ,,	,,
d	106	4·5	4763	0·0100	0·0069	−0·0031	0·2 ,,	,,
a	33	3·3	4957	0·0105	0·0074	−0·0031	0·2 ,,	,,

The levels decrease from top to bottom of the table.

The discrepancies are large and positive for high-level lines, and again large and negative for lines at low level, and small for lines at intermediate levels. In the last column are the downward and upward velocities that would bring observations and prediction into agreement; especially striking are the results from lines of group *a*, the most reliable lines in the iron spectrum. The total red displacement cannot be attributed to downward velocities, as the wave-lengths at the limb would be shorter than at the centre of the disc, while in fact they are on the whole longer.

If the half-effect of Larmor be used, the resulting velocities, even at the lowest levels, are downward. Anomalous refraction requires, according to Julius, the displacements for strong lines to be the least, while in fact they are the largest of all.

Considering the situation as I now see it, displacements predicted by generalised relativity, combined with small Doppler displacements, offer the most probable interpretation of the differences between wave-lengths at the centre of the sun's disc and the arc *in vacuo*. The continued existence of currents necessary to bring the observed and the calculated displacements into agreement for lines of very high and very low levels present some questions that will be discussed in *Contributions from Mount Wilson Observatory*, where fuller data will appear.

There remains to be considered the further displacements to the red at the edge of the sun's disc. The increased length of path at the limb through layers under supposedly great pressure was suggested by Halm, Fabry and Buisson, and Adams as the dominating cause of limb-centre shift, but an explanation based upon pressure is no longer tenable with pressures of a small fraction of an atmosphere.

Professor Julius has called attention to the fact that the refractive power is greater on the red than on the violet side of an absorption line by twice the refractive power of the solar atmosphere. He would ascribe the displacement between limb and centre to an asymmetry which he says depends upon anomalous refraction and scattering in irregular density-gradients. Aside from the consideration that persistently irregular density-gradients are improbable, the very low pressure would hardly furnish the steep gradients required for anomalous refraction.

There remains the effect of selective molecular scattering in accordance with the Rayleigh-Schuster formulæ. The coefficient of scattering increases as the square of the refractive power and as the resultant refractive power is a little greater on the red than on the violet side of the line, there is a differential effect tending to widen the line on the red edge. The short path in layers of low density at the centre of the disc would account for the absence of differential widening at the centre, but the greatly lengthened path at the limb through the lower layers would furnish conditions favourable for the differential effect.

The suggested interpretation of limb-shift follows the line of reasoning used by Halm, Fabry and Buisson, and Adams, but involves, in place of pressure, differential scattering in paths of increased length, and requires neither large nor unusual density-gradients.

The conclusion is that three major causes are producing the regular

differences between solar and terrestrial wave-lengths, and that it is possible to disentangle their effects. The causes appear to be the slowing up of the atomic clock in the sun to an amount predicted by the theory of generalised relativity, radial velocities of moderate cosmic magnitude and in probable directions, and differential scattering in the longer paths traversed through the solar atmosphere by light coming from the limb of the sun. The first obtains for all lines in all parts of the sun, the second appears regularly and continuously, downward at very high and upward at very low levels, while the third manifests itself in the so-called limb effect.

Solar Faculæ—Their Occurrence and Distribution in Latitude. (Plate 3.)

(*Communicated by the Astronomer Royal.*)

The Greenwich Photo-heliographic Results contain measures of the solar faculæ made since the year 1874, when the photographic record of the sun's surface was inaugurated at the Royal Observatory, Greenwich. Since then, nearly 70,000 measures of faculæ have been made and published in the *Daily Results* of the Greenwich volumes, and their mean daily area, rotation by rotation and also for each year as a whole, has been given with the figures for the sun-spots which were suitable for more detailed measurement. As an index of the solar cycle of eleven years, the faculæ appear to follow the spots very closely; indeed, they give a slightly smoother curve than that derived from the spots themselves. This is somewhat remarkable, since the faculæ are usually visible only for about half the time they are actually in transit across the sun's disc. Moreover, owing to their nature, estimates of area are always difficult, and their measurement, necessarily near the sun's limbs, introduces some uncertainty, especially in longtitude, owing to the effect of foreshortening. Partly on account of these inherent difficulties, their distribution on the sun's surface for successive cycles has never been strictly defined, but their close relationship with spots has been frequently commented upon, and their appearance beyond the spot zones has also been recorded at times by different observers.

It was therefore a simple and useful piece of work to plot every observation of a group of faculæ for each rotation of the sun from 1874–1917 according to its latitude on the sun's surface, and roughly to indicate its area by a certain thickness of line. The results of the plot are reproduced in the accompanying diagram. Whenever a facula has appeared, whether connected with a spot group or seen by itself, it has been represented by a line 1° of solar latitude in length; while the thicker the line the larger the area represented. The latitude of the chief faculæ zones is but little affected by any error of foreshortening, and the graph should therefore indicate with very fair precision the distribution of the faculæ as a whole during a solar cycle.

Distribution of Faculæ in Heliographic Latitude.

The lower diagram shows the mean daily area of faculæ for each rotation of the sun from 1874–1917.

An inspection of the diagram brings out the following points :—

(1) The centres of the chief zones of the faculæ have a well defined progression with the solar cycle of about eleven years. They show in this respect considerable accordance with the latitude progression of sun-spots so clearly indicated in Mr. E. W. Maunder's diagram ("Distribution of Sun-spots in Heliographic Latitude, 1874–1913," *M.N.R.A.S.*, 74, Plate 2), which suggested the construction of a similar one for the faculæ.

(2) As compared with the spot zones, however, the corresponding faculæ zones are on the average about 15° broader. The extension is mainly polewards, and from the shading of the diagram it is seen that it is made up entirely of small areas of faculæ, the largest areas being always confined to the region of the spots.

(3) The largest individual areas occur most frequently just about the maximum of spot activity. For two or three years after maximum, although the zones are broader, they are less intense ; *i.e.* the average size of the groups of faculæ is smaller, although they may be more numerous.

(4) There is a zone of polar faculæ in both hemispheres showing slight concentration about latitude 70°. The polar faculæ differ from those seen in lower latitudes. They appear invariably as small, short-lived, detached flecks. Their measurement is necessarily somewhat dependent upon the successive presentation of the sun's north and south poles to the earth in the course of the year. Their appearance seems to be somewhat erratic, and shows no pronounced relationship with the solar cycle. From the material at present available, they do not appear to be in any way associated with the polar prominences, which reach latitude 70° about the maximum years of the cycle.

The next matter was to determine the distribution of the faculæ *unassociated* with sun-spots. This involved the work of tracing the complete history of each area from its first appearance to its extinction. Ledgers were therefore prepared on the same principle as the spot ledgers, and, although in many cases the work of identification was very involved (indeed impossible near the maximum), eight years from the present cycle were completely analysed. Generalising from these years one can say :—

(5) Excepting the polar faculæ, which are relatively few, the average latitude of faculæ unassociated with sun-spots varies in the same general way as the sun-spot zones, but they are broader, as mentioned in paragraph (2).

(6) In character, the faculæ unassociated with sun-spots are small, faint areas lasting at the most for two months. This characteristic offers a striking contrast to the faculæ connected with sun-spots, which develop in a few days into bright and compact masses often covering a great area. These after about two months become faint and more scattered, but they can nearly always be discriminated from those which have at no time been seen with sun-spots. With large spot disturbances, the faculæ can be recognised frequently for several months.

This characteristic was found by actual examination of the photographs of nearly two thousand areas of faculæ with which no spots were recorded. About 650 of these areas were seen for the first time at the west limb of the sun (*i.e.* they had actually formed on the visible hemisphere); not a single one of these was *large*, *bright*, and *compact*.

(7) For the years 1913–17 and 1920–22, taken as representative of the complete cycle, the mean percentage area of faculæ unconnected with spots as compared with the total area of all faculæ is about 10 per cent. This figure increases to about 30 per cent. during the minimum years and falls to 6 per cent. when the sun is active.

If the numbers of areas are considered, the proportion is about 60 per cent. at minimum and 25 per cent. at maximum. This corroborates the statement in the previous paragraph that the average size of the faculæ unassociated with sun-spots is much smaller than those associated with them.

(8) During the course of the preparation of this paper many hundreds of faculous regions have been examined on the photographs. It is perhaps worth while to record also the following observations, some of which struck the examiner of the photographs, and others which have been derived from the figures.

(*a*) An area of very bright, compact faculæ indicates that either a spot group has been very recently connected with it or that one will appear within a few hours.

(*b*) No example has been recorded of a spot seen near the sun's limbs without attendant faculæ. There were many instances when faculæ preceded the appearance of a spot, in some cases by several hours, in others by several days. In a very large number of cases the two phenomena developed almost simultaneously. Even the smallest spot groups appear to have attendant faculæ. During the years 1914–17, 660 very small, ephemeral spots were measured. In only 60 cases could accompanying faculæ not be identified when the region approached the sun's limb where faculæ were seen when present. As the 60 one-day spots considered were, with seven exceptions, within 30° of the central meridian, it is reasonable to assume that faculæ may have accompanied the spots, but were short-lived.

(*c*) The duration of faculæ connected with a spot group is on the average at least three times the length of the accompanying spots, but the proportion is a very variable one. Generally speaking, the faculæ of large spot groups as compared with those of smaller groups do not last proportionally as long.

(*d*) The faculæ frequently act as a connecting link between successive spot disturbances in the same region. Near the maximum of the solar cycle, some of these centres have been traced without intermission for over six months and a few for nearly a year.

(*e*) Faculæ tend to cluster around the preceding and following members of a stream of spots of normal type, or "bipolar" type according to the Mt. Wilson classification. In several instances these two distinct centres have been seen long after the spots have disappeared. The two centres have the same tendency to separate from each other in longitude as shown by the leading and following spots of a stream.

(*f*) Faculæ frequently appear in streaks roughly at right angles to the direction of the sun's rotation, and have a strong tendency to spread from a spot disturbance for several degrees in latitude. This feature contrasts with the spots themselves which invariably stream out in longitude, whilst there is little trend in the direction of latitude.

(*g*) The faculæ change form rapidly, and individual features can seldom be recognised at successive appearances at the sun's limbs. The groups of faculæ considered as entities are, however, more stable than groups of spots which are frequently reduced to a single component (usually the leader) within about fifteen days. The regularity of statistics of sun-spots is largely disturbed by the fact that we have an imperfect record of the spots owing to the sun's rotation. The longer life of the faculæ tends to smooth this, for sun-spots which form and disappear on the hemisphere turned away from the earth will usually be marked by the attendant faculæ which survive them. In fact, an examination of the type of faculæ (see paragraph 6), seen for the first time at the east limb of the sun, will be a reliable index of spot activity on the invisible hemisphere of which there is no other record.

(9) Finally, the investigation of the faculæ, as outlined above, has brought out more strongly than was anticipated the very close connection between them and the sun-spots. Excluding the polar faculæ and about 10 per cent. of the total area measured in the spot zones, faculæ are always related to spots. There are no spots without faculæ, and no extensive areas of faculæ without spots. The importance of continuing the observations of faculæ is emphasised, inasmuch as they prove, in spite of their difficulty of measurement, a very accurate index of the progress of the solar cycle, both in their occurrence and their distribution on the sun's surface.

Royal Observatory, Greenwich:
1923 *November* 21.

Micrometrical Measures of Double Stars.

By Rev. T. E. R. Phillips, M.A.

Star.	1925. R.A.	1925. Dec.	Mags.	Position Angle.	Distance.	No. of Nights.	Date.
	h m	° ′		°	″		
Σ 3056	0 0·8	+33 52	7·4, 7·4	52·0	12·75	1	1921·847
3060	0 2·2	17 40	8·5, 8·7	121·3	3·54	2	·907
24	0 14·6	25 43	7·2, 8·0	247·6	5·06	2	·922
OΣ 12 (λ Cassiop.)	0 27·6	54 6	5·5, 5·8	158·8	0·52	3	·771
				158·9	0·60	3	1923·063
Σ 59	0 43·6	51 2	7·2, 8·1	326·4	2·15	2	·990
60 (η Cassiop.)	0 44·5	57 25	3·7, 7·4	259·5	7·32	4	1921·676
				261·9	7·56	3	1922·623
61	0 45·8	27 18	6·0, 6·0	115·9	4·18	3	1921·909

Star.	1925. R.A.	Dec.	Mags.	Position Angle.	Distance.	No. of Nights.	Date.
	h m	° ′		°	″		
Σ 73 (36 Androm.)	0 50·9	+23 13	6·1, 6·7	56·1	0·78	3	·713
113 (42 Ceti)	1 16·0	− 0 54	6·4, 7·4	358·8	1·70	3	·677
138	1 32·1	+ 7 16	7·3, 7·3	42·0	1·85	1	1923·045
186	1 52·0	1 28	7·2, 7·2	40·8	1·26	5	1921·732
202 (α Piscium)	1 58·2	2 24	2·8, 3·9	311·0	2·52	3	·744
OΣ 38 (γ^2 Androm.)	1 59·3	41 58	5·4, 6·6	110·5	0·56	3	·651
				109·7	0·50	3	1922·792
Σ 240	2 13·0	23 41	7·7, 8·2	49·6	4·65	3	1921·784
314	2 47·5	52 41	6·9, 7·1	304·3	1·61	3	·668
OΣ 43	2 36·3	26 17	7·2, 8·8	32·0	0·96	3	1921·765
Σ 333 (ε Arietis)	2 54·9	21 3	5·7, 6·0	203·0	1·39	3	·656
346 (52 Arietis) AC	3 1·0	24 58	6·0, 6·0	356·7	5·25	3	·699
425	3 35·4	33 53	7·3, 7·3	86·3	2·26	3	·809
670	5 12·4	+18 21	7·7, 8·2	165·2	2·28	2	·825
Dawes 5 (η Orionis)	5 20·7	− 2 28	3·8, 4·8	79·7	1·41	3	·763
Sirius	6 41·8	−16 37	−1·6, 8·4	63·4	11·22	3	1922·062
				62·4	11·21	4	1923·053
Σ 948 (12 Lyncis)	6 39·6	+59 32	5·2, 6·1	108·2	1·78	2	1921·669
1110 (Castor)	7 29·8	32 3	2·0, 2·8	214·8	4·68	3	·922
				214·7	4·59	4	1922·996
1126	7 36·1	5 24	6·4, 6·7	149·6	1·08	5	1923·025
1196 (ζ Cancri)	8 7·9	17 53	5·6, 6·3	256·9	0·66	4	·028
1338	9 16·3	38 31	7·0, 7·2	183·2	1·76	1	1923·133
1356 (ω Leonis)	9 24·4	9 23	5·9, 6·7	130·4	1·02	2	1922·275
1424 (γ Leonis)	10 15·8	20 13	2·6, 3·8	116·8	3·69	5	·237
				117·9	3·79	3	1923·037
1507	11 2·2	7 26	8·2, 10·3	165·3	8·10	3	1921·243
1639 (68 Comæ)	12 20·7	26 0	6·7, 7·9	337·5	0·61	3	1923·092
1643	12 23·5	27 27	8·4, 8·7	28·2	2·28	3	·092
1647	12 26·7	10 8	7·5, 7·8	47·6	1·50	2	·068
1687 (35 Comæ)	12 49·6	21 39	5·0, 7·8	96·7	0·99	2	1921·252
1785	13 45·9	27 21	7·2, 7·5	26·8	1·17	2	1923·235
1864 (π Bootis)	14 37·2	16 44	4·9, 6·0	104·5	5·69	3	1921·560
1865 (ζ Bootis)	14 37·6	14 3	3·5, 3·9	136·1	0·76	10	·536
				136·6	0·92	6	1922·634
				137·6	0·81	3	1923·142
1877 (ε Bootis)	14 41·7	27 23	2·7, 5·1	329·9	2·73	4	1921·488
1888 (ξ Bootis)	14 47·9	19 25	4·8, 6·8	58·0	2·63	6	·369
				53·2	2·68	10	1922·526
				48·8	2·97	2	1923·192

Star.	1925. R.A.	1925. Dec.	Mags.	Position Angle.	Distance.	No. of Nights.	Date.
	h m	° ′		°	″		
Σ 1932	15 15·1	27 6	5·6, 6·1	16·9	0·57	2	1921·702
1937 (η Cor. B.)	15 20·1	30 33	5·6, 6·1	101·7	0·53	6	·624
1938 (μ^2 Bootis)	15 21·7	37 36	6·7, 7·3	47·1	1·56	4	·679
OΣ 298	15 33·4	40 3	7·4, 7·7	209·4	0·67	3	1922·577
2054	16 22·8	61 51	5·7, 6·9	355·7	1·16	3	·633
312 (η Draconis)	16 23·0	61 40	2·1, 8·1	141·7	5·28	3	1921·667
2055 (λ Ophiuchi)	16 27·1	2 9	4·0, 6·1	82·6	0·82	3	·597
2059	16 28·1	38 14	8·2, 8·3	202·6	0·92	3	·619
2084 (ζ Herculis)	16 38·5	31 44	3·0, 6·0	79·5	1·59	3	1922·636
2140 (α Herculis)	17 11·2	14 28	3·0, 6·1	111·3	4·58	4	1921·646
2161 (ρ Herculis)	17 21·1	37 12	4·0, 5·1	313·3	3·85	3	·662
2264 (95 Herculis)	17 58·3	21 36	4·9, 4·9	257·9	6·14	3	·600
2272 (70 Ophiuchi)	18 1·7	2 31	4·3, 6·0	131·3	5·58	4	·559
				130·1	5·50	6	1922·572
				127·9	5·69	3	1923·649
2382 (ϵ^1 Lyræ)	18 41·9	39 35	5·1, 6·0	7·2	2·95	4	1921·630
2383 (ϵ^2 Lyræ)	18 41·9	39 32	5·4, 5·4	116·8	2·32	4	·630
2481 (AB)*	19 8·6	38 40	8·0, 8·0	215·6	4·33	3	·589
2579 (δ Cygni)	19 42·6	44 57	3·0, 7·9	274·5	1·83	3	·610
				276·4	1·89	4	1922·610
2583 (π Aquilæ)	19 45·2	11 38	6·0, 6·8	113·2	1·59	4	1921·682
OΣ 395 (16 Vulpec.)	19 58·8	24 43	5·8, 6·2	105·1	0·81	3	·698
413 (λ Cygni)	20 44 5	36 13	4·8, 6·1	47·9	0 64	3	·696
Σ 2760	21 3·7	33 50	7·3, 8·1	228·4	4·24	3	·591
2822 (μ Cygni)	21 40 8	+28 24	4·7, 6·1	136·4	1·42	3	·687
2909 (ζ Aquarii)	22 25·0	− 0 24	4·4, 4·6	302·8	2·71	3	·624
OΣ 483 (52 Pegasi)	22 55·4	+11 20	6·0, 7·5	238·3	0·94	3	1922·608
Σ 3050	23 55·7	+33 19	6·5, 6·6	227·9	1·99	2	·651

* The star B seen as a close double with discs in contact, P.A. = 84°·8.

Headley Rectory, Epsom:
1923 *December* 7.

Erratum.

M.N. 84, 22, Table, line K2, *for* OY *read* OY^2.